
Efficient Algorithms and Datastructures II

Aufgabe 1 (10 Punkte)

Consider the following maximization version of the 3-Dimensional Matching Problem. Given disjoint sets X, Y, Z and a set $T \subseteq X \times Y \times Z$ of ordered triples, a subset $M \subseteq T$ is a 3-dimensional matching if each element of $X \cup Y \cup Z$ is contained in at most one of these triples. The Maximum 3-Dimensional Matching Problem is to find a 3-dimensional matching M of maximum cardinality.

Give a polynomial-time algorithm that finds a 3-dimensional matching of size at least $\frac{1}{3}$ times the maximum possible size.

Aufgabe 2 (10 Punkte)

We are given k stretchable bags b_1, \dots, b_k and n items a_1, \dots, a_n with weights w_1, \dots, w_n and volume v_1, \dots, v_n respectively, such that $w_i, v_i \leq 1$ and $\sum_{i=1}^n w_i = k = \sum_{i=1}^n v_i$. We say that a packing of the n items in the k bags is an (α, β) -packing if each bag is filled with weight $\leq \alpha$ and volume $\leq \beta$. Give an efficient algorithm for obtaining a $(3, 3)$ -packing.

Aufgabe 3 (10 Punkte)

You have a system that consists of m slow machines and k fast machines. The fast machines can perform twice as much work per unit time as the slow machines. You are given a set of n jobs; job i takes time t_i to process on a slow machine and time $\frac{1}{2}t_i$ to process on a fast machine. You want to assign each job to a machine so as to minimize the makespan - the makespan is the maximum, over all machines, of the total processing time of jobs assigned to that machine.

Give a polynomial-time algorithm that produces an assignment of jobs to machines with a makespan that is at most three times the optimum.