Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
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Spring Semester 2014
Problem set 4
April 30, 2014

Online and approximation algorithms

Due May 07, 2014 before class!

Exercise 1 (RMARK - 10 points)

Recall that RMARK is the randomized online paging algorithm that works as follows: Initially all pages are unmarked. Whenever a page is requested it becomes marked. When a page is brought into the cache it replaces a randomly and uniformly chosen page from the set of unmarked pages that are in the cache. When all pages in the cache are marked, we unmark all pages.
We saw in the lecture, that RMARK is $2H_k$-competitive. Prove that RMARK is H_k-competitive against an oblivious opponent when the total number of pages N is $k + 1$.

Exercise 2 (RMARK II - 10 points)

Show that RMARK is not in general H_k-competitive.
Hint: $k = 2$, $N = 4$

Exercise 3 (Randomized ski rental (upper bound) - 10 points)

Show that there exists a randomized algorithm for the ski rental problem that achieves a competitive ratio that is better than 2 against an oblivious adversary.

Exercise 4 (Randomized ski rental (lower bound) - 10 points)

Use Yao's Principle to prove that the competitive ratio of all randomized algorithms for the ski rental problem is lower bounded by $\frac{4}{3}$.