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## Efficient Algorithms and Datastructures II

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### Aufgabe 1 (10 Punkte)

In the maximum directed cut problem, we are given a directed graph  $G = (V, A)$ , and non-negative weights  $w_{ij} \geq 0, \forall (i, j) \in A$ . The goal is to partition  $V$  into 2 parts  $U$  and  $W$  so as to maximize the total weights of the arcs going from  $U$  to  $W$ . (we say that  $(i, j)$  goes from  $U$  to  $W$  if  $i \in U$  and  $j \in W$ ). Give a randomized  $\frac{1}{4}$  approximation algorithm for this problem.

### Aufgabe 2 (10 Punkte)

- (a) In the maximum  $k$ -cut problem, we are given an undirected graph  $G = (V, E)$ , and non-negative weights  $w_{ij} \geq 0, \forall (i, j) \in E$ . The goal is to partition the vertex set  $V$  into  $k$  parts  $V_1, \dots, V_k$  so as to maximize the weights of all edges whose endpoints are in different parts (i.e.,  $\max_{(i,j) \in E: i \in V_a, j \in V_b, a \neq b} w_{ij}$ ). Give a randomized  $\frac{k-1}{k}$  approximation algorithm for the maximum  $k$ -cut problem.
- (b) Derandomize the above algorithm.

### Aufgabe 3 (10 Punkte)

- (a) Show that for every  $0 < \epsilon < 1$ ,

$$PCP_{\frac{2}{3}, \frac{1}{3}}[r, q] \subseteq PCP_{1-\epsilon, \epsilon}[O(r \cdot \log(1/\epsilon)), O(q \cdot \log(1/\epsilon))]$$

- (b) Show that for any  $r$  and  $\epsilon > 0$ ,

$$PCP_{\frac{2}{3}, \frac{1}{3}}[r, O(1)] \subseteq PCP_{1-\epsilon, \epsilon}[r, \text{poly}(1/\epsilon)]$$