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Summersemester 2014
Übungsblatt 7
June 3, 2014

Efficient Algorithms and Datastructures II

Aufgabe 1 (10 Punkte)

Given a directed graph $G = (V, E)$, we want to find a maximum cardinality set of edges $E' \subseteq E$ such that the graph $G = (V, E')$ is acyclic. Give a factor $\frac{1}{2}$ algorithm for this problem.
(*Hint*: Arbitrarily number the vertices and pick one of the two sets of edges - the forward edges and the backward edges)

Aufgabe 2 (10 Punkte)

Given an undirected graph $G = (V, E)$, a valid k-coloring is an assignment of its vertices to k colors such that the two endpoints of each edge receive distinct colors. The minimum vertex coloring problem is to find the minimum k such that G is k-colorable.

1. Give an algorithm for coloring G with $\Delta+1$ colors, where Δ is the maximum degree of a vertex in G.

2. Give an algorithm for coloring a 3-colorable graph with $O(\sqrt{n})$ colors.

Aufgabe 3 (10 Punkte)

1. Prove that any "vertex" point of the LP

$$\begin{array}{lrcll} \text{minimize} & \sum_{i \in V} w_i x_i & & & \\ \text{subject to} & x_i + x_j & \geq & 1 & \forall (i,j) \in E \\ & x_i & \geq & 0 & \forall i \in V \end{array}$$

   has the property that $x_i \in \left\{0, \frac{1}{2}, 1\right\} \forall i \in V$.

2. Give a $\frac{3}{2}$ - approximation algorithm for the vertex cover problem when the input graph is planar. Use the facts that we can find an optimal "vertex" point in polynomial time and there is a polynomial time algorithm to 4-color any planar graph.