Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
Prof. Dr. Harald Räcke
Chris Pinkau

Winter Semester 2013/14
Problem Set 10
January 21th, 2014

Parallel Algorithms

Due date: January 28th, 2014 **before** *class!*

Problem 1 (10 Points)

Consider greedy routing on the $n \times n$ mesh.

1. Show that the worst-case buffer size can be as high as $\frac{3}{2}n - \mathcal{O}(1)$ when a node can only forward one packet in every step.
2. Show that the worst-case buffer size can be as high as $\frac{2}{3}n - \mathcal{O}(1)$ when a node can forward any number of packets in every step (bounded by the number of incident edges of course).

Argue that these bounds are tight, i.e. there are no cases where the buffer size exceeds these bounds.

Problem 2 (10 Points)

Recall the analysis presented in the lecture about randomized oblivious routing on the mesh where, for a given source-target pair (s, t), we found a submesh M_k of size $2^k \times 2^k$ with s in one corner and $t \in M_k$. The analysis covered only the case if M_k is entirely contained in the original mesh. Describe an approach for the analysis when M_k is not entirely contained in the original mesh.
Hint: Observe that the analysis on a torus would be straightforward.

Problem 3 (10 Points)

A randomized oblivious routing scheme is *α-competitive* if, for every concurrent multicommodity flow problem P with given demands, it holds for the congestion C that $C \leq \alpha \cdot C_{opt}(P)$.

1. Consider an n-node ring network and prove that no oblivious routing scheme can have a competitive ratio of less than $2\left(1 - \frac{1}{n}\right)$.
   Hint: Consider the case where every node sends one unit of flow to its right neighbor and receives one unit from its left neighbor, and the case where only a single node sends one unit of flow to one of its neighbors.
2. Show that in an n-node ring network shortest-path-routing is 2-competitive.