Technische Universität München
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Assignment 10
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Effiziente Algorithmen und Datenstrukturen I

Question 1 (10 Punkte)

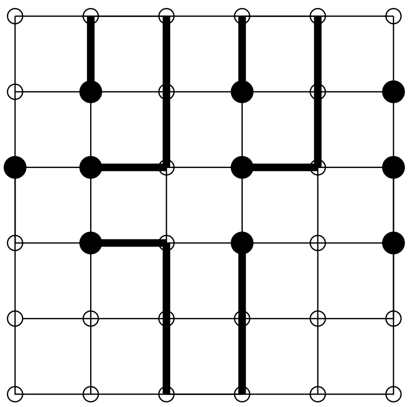
Show that a maximum flow in a network $G = (V, E)$ can always be found by a sequence of at most $|E|$ augmenting paths.
(*Hint:* Determine the paths after finding the maximum flow.)

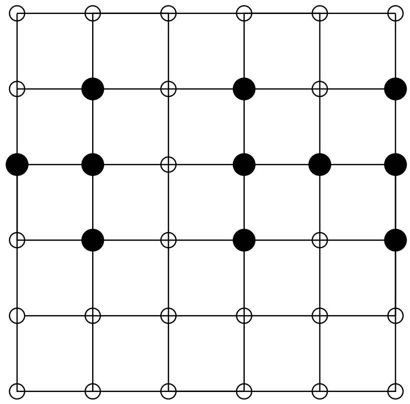
Question 2 (10 Punkte)

Consider a flow network in which vertices, as well as edges, have capacities. That is, the total positive flow entering any given vertex is subject to a capacity constraint. Show that determining the maximum flow in a network with edge and vertex capacities can be reduced to an ordinary maximum-flow problem on a flow network of comparable size.

Question 3 (10 Punkte)

An $n \times n$ grid is an undirected graph consisting of n rows and n columns of vertices, as shown in the figures below. We denote the vertex in the ith row and the jth column by (i, j). All vertices in a grid have exactly four neighbors, except for the boundary vertices, which are the points (i, j) for which $i = 1$, $i = \text{n}$, $j = 1$, or $j = \text{n}$. Given $m \le n^2$ starting points $(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)$ in the grid, the escape problem is to determine whether or not there are m vertex-disjoint paths from the starting points to any m different points on the boundary. For example, the first grid below has an escape, but the second grid does not. Starting points are shown in black.





Describe an efficient algorithm to solve the escape problem, and analyze its running time. You may use the result of a previous solution in designing your algorithm.

Question 4 (10 Punkte)

Let $G = (V, E)$ be a flow network with source s, sink t, and integer capacities. Suppose that we are given a maximum flow in G.

(a) Suppose that the capacity of a single edge $(u, v) \in E$ is increased by 1. Give an $O(V + E)$ time algorithm to update the maximum flow.

(b) Suppose that the capacity of a single edge $(u, v) \in E$ is decreased by 1. Give an $O(V + E)$ time algorithm to update the maximum flow.