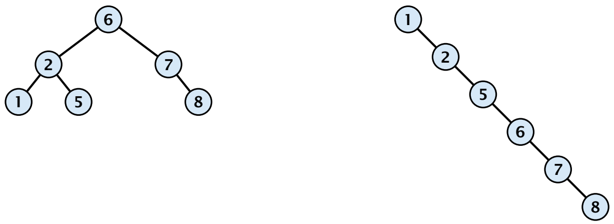
7.1 Binary Search Trees

An (internal) binary search tree stores the elements in a binary tree. Each tree-node corresponds to an element. All elements in the left sub-tree of a node v have a smaller key-value than $\text{key}[v]$ and elements in the right sub-tree have a larger-key value. We assume that all key-values are different.

(External Search Trees store objects only at leaf-vertices)

Examples:



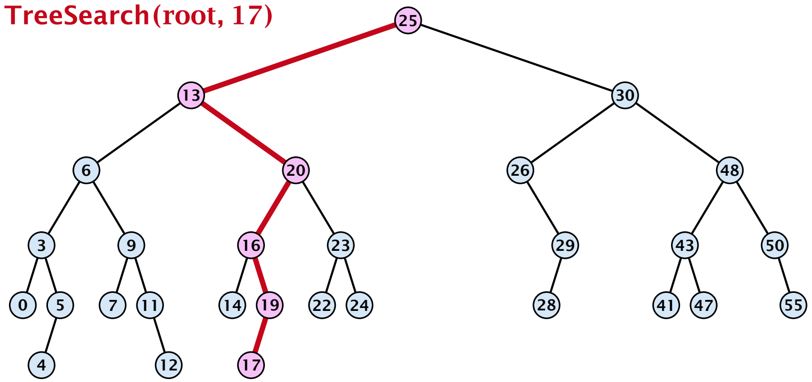
7.1 Binary Search Trees

We consider the following operations on binary search trees. Note that this is a super-set of the dictionary-operations.

- $T.\text{insert}(x)$
- $T.\text{delete}(x)$
- $T.\text{search}(k)$
- $T.\text{successor}(x)$
- $T.\text{predecessor}(x)$
- $T.\text{minimum}()$
- $T.\text{maximum}()$

Binary Search Trees: Searching

TreeSearch(root, 17)

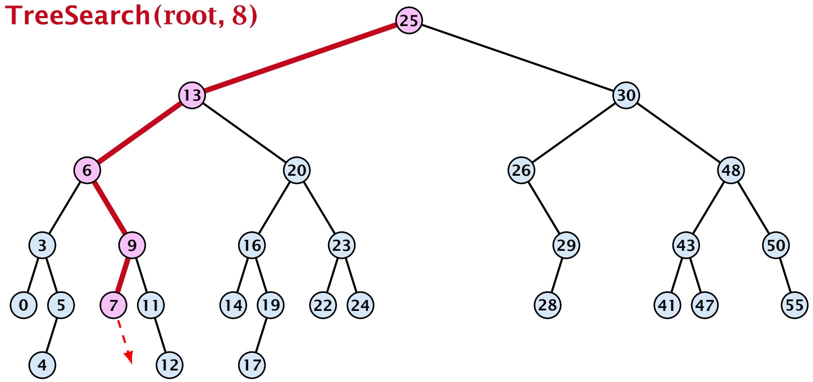


Algorithm 5 TreeSearch(x, k)

if $x = \text{null}$ **or** $k = \text{key}[x]$ **return** x
if $k < \text{key}[x]$ **return** TreeSearch$(\text{left}[x], k)$
else return TreeSearch$(\text{right}[x], k)$

Binary Search Trees: Searching

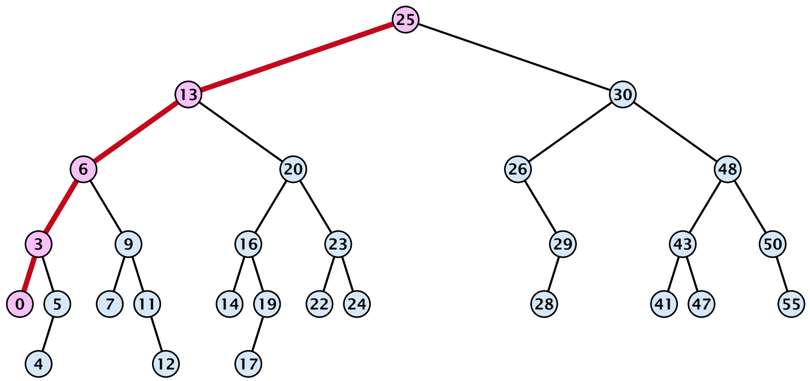
TreeSearch(root, 8)



Algorithm 5 TreeSearch(x, k)

if $x = \text{null}$ **or** $k = \text{key}[x]$ **return** x
if $k < \text{key}[x]$ **return** TreeSearch$(\text{left}[x], k)$
else return TreeSearch$(\text{right}[x], k)$

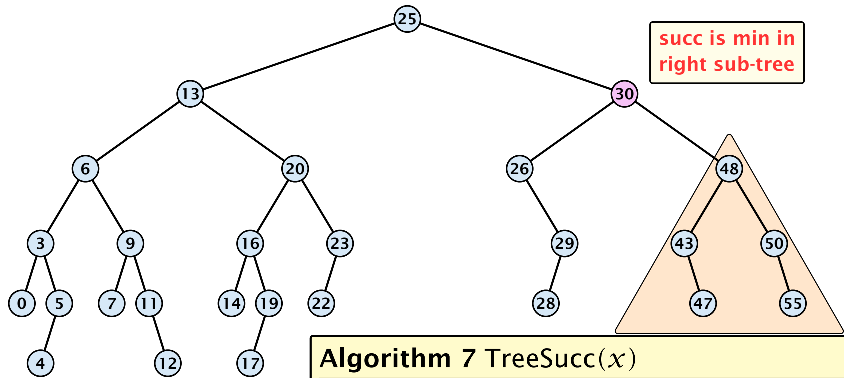
Binary Search Trees: Minimum



Algorithm 6 TreeMin(x)

1: **if** x = null **or** left[x] = null **return** x
2: **return** TreeMin(left[x])

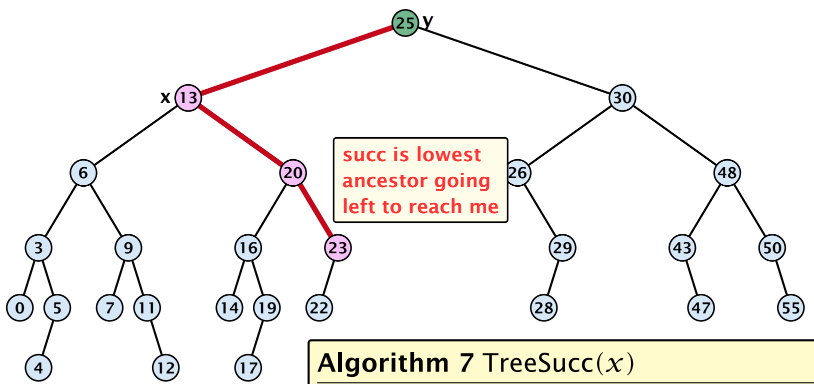
Binary Search Trees: Successor



Algorithm 7 TreeSucc(x)

1: **if** right[x] $\neq$ null **return** TreeMin(right[x])
2: $y \leftarrow$ parent[x]
3: **while** $y \neq$ null **and** x = right[y] **do**
4: $x \leftarrow y$; $y \leftarrow$ parent[x]
5: **return** y;

Binary Search Trees: Successor



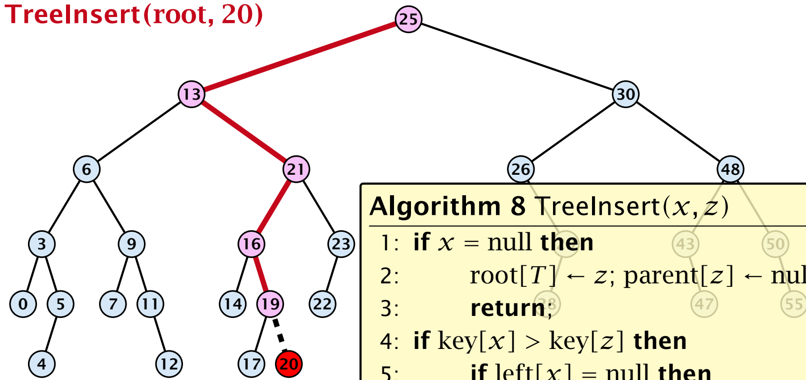
Algorithm 7 TreeSucc(x)

1: **if** right[x] $\neq$ null **return** TreeMin(right[x])
2: $y \leftarrow$ parent[x]
3: **while** $y \neq$ null **and** x = right[y] **do**
4: $x \leftarrow y$; $y \leftarrow$ parent[x]
5: **return** y;

Binary Search Trees: Insert

Insert element **not** in the tree.

TreeInsert(root, 20)

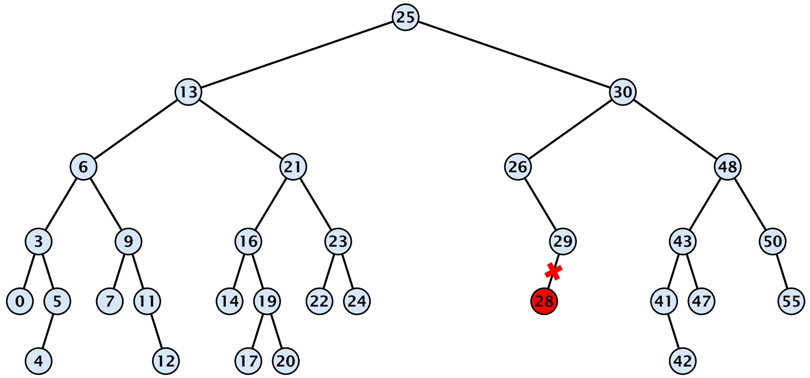


Search for z. At some point the search stops at a null-pointer. This is the place to insert z.

Algorithm 8 TreeInsert(x, z)

1: **if** x = null **then**
2: root[T] $\leftarrow z$; parent[z] $\leftarrow$ null;
3: **return**;
4: **if** key[x] > key[z] **then**
5: **if** left[x] = null **then**
6: left[x] $\leftarrow z$; parent[z] $\leftarrow x$;
7: **else** TreeInsert(left[x], z);
8: **else**
9: **if** right[x] = null **then**
10: right[x] $\leftarrow z$; parent[z] $\leftarrow x$;
11: **else** TreeInsert(right[x], z);

Binary Search Trees: Delete

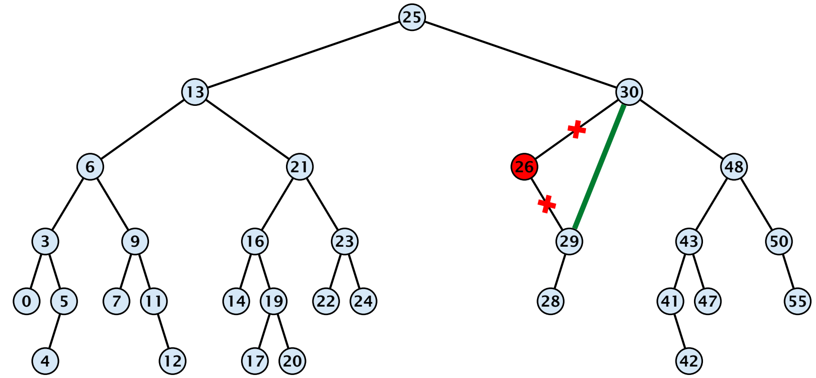


Case 1:

Element does not have any children

- Simply go to the parent and set the corresponding pointer to null.

Binary Search Trees: Delete

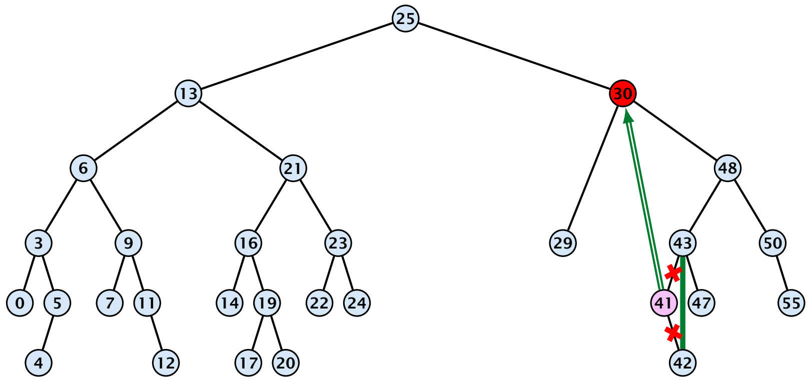


Case 2:

Element has exactly one child

- Splice the element out of the tree by connecting its parent to its successor.

Binary Search Trees: Delete



Case 3:

Element has two children

- Find the successor of the element
- Splice successor out of the tree
- Replace content of element by content of successor

Binary Search Trees: Delete

Algorithm 9 TreeDelete(z)

```
if left[z] = null or right[z] = null
    then y ← z else y ← TreeSucc(z);          select y to splice out
if left[y] ≠ null
    then x ← left[y] else x ← right[y];  x is child of y (or null)
if x ≠ null then parent[x] ← parent[y];       parent[x] is correct
if parent[y] = null then                  ⎫
    root[T] ← x                           ⎪
else                                      ⎪
    if y = left[parent[y]] then           ⎬ fix pointer to x
        left[parent[y]] ← x               ⎪
    else                                  ⎪
        right[parent[y]] ← x              ⎭
if y ≠ z then copy y-data to z
```

Balanced Binary Search Trees

All operations on a binary search tree can be performed in time $\mathcal{O}(h)$, where h denotes the height of the tree.

However the height of the tree may become as large as $\Theta(n)$.

Balanced Binary Search Trees
With each insert- and delete-operation perform local adjustments to guarantee a height of $\mathcal{O}(\log n)$.

AVL-trees, Red-black trees, Scapegoat trees, 2-3 trees, B-trees, AA trees, Treaps

similar: SPLAY trees.

Binary Search Trees (BSTs)

Bibliography

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[CLRS90] Thomas H. Cormen, Charles E. Leiserson, Ron L. Rivest, Clifford Stein:
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Binary search trees can be found in every standard text book. For example Chapter 7.1 in [MS08] and Chapter 12 in [CLRS90].