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How do you measure?

- Implementing and testing on representative inputs
  - How do you choose your inputs?
  - May be very time-consuming.
  - Very reliable results if done correctly.
  - Results only hold for a specific machine and for a specific set of inputs.
- Theoretical analysis in a specific model of computation.
  - Gives [illegible] like "this algorithm always runs in time $\mathcal{O}(n^2)$".
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Model of Computation

How to measure performance

- Calculate running time and storage space etc. on a simplified, idealized model of computation, e.g. Random Access Machine (RAM), Turing Machine (TM), ...
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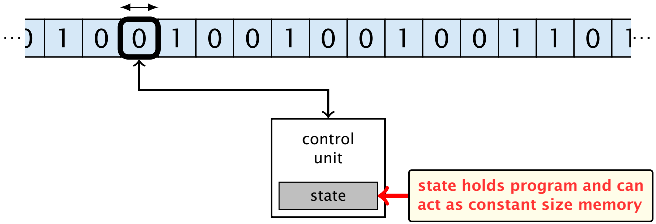
Turing Machine

- Very simple model of computation.
- Only the “current” memory location can be altered.
- Very good model for discussing computabiliy, or polynomial vs. exponential time.
- Some simple problems like recognizing whether input is of the form xx, where x is a string, have quadratic lower bound.

$\Longrightarrow$ **Not a good model for developing efficient algorithms.**



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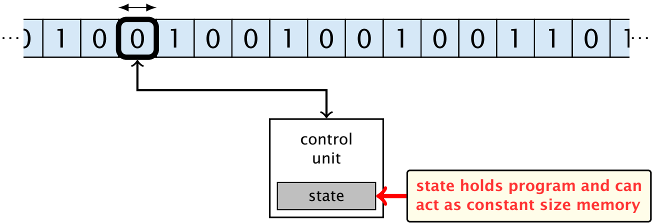

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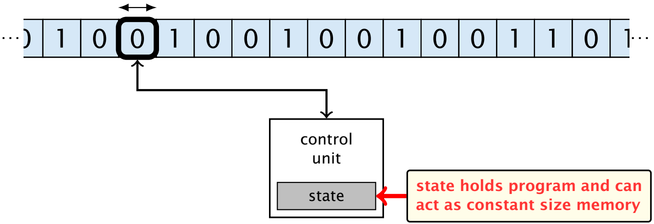

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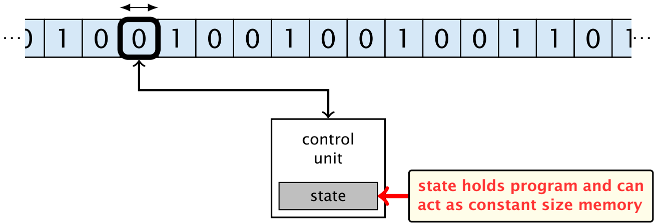

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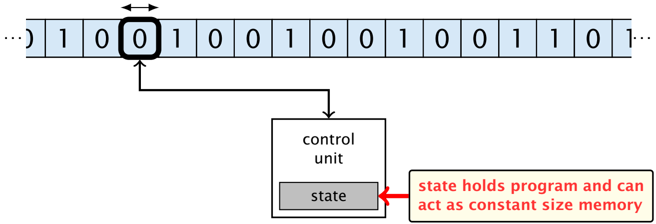
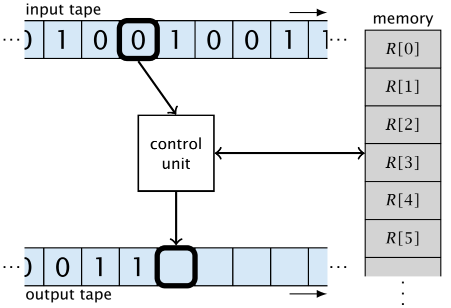


Random Access Machine (RAM)

- Input tape and output tape (sequences of zeros and ones; unbounded length).
- Memory unit: infinite but countable number of registers $R[0], R[1], R[2], \ldots$.
- Registers hold integers.
- Indirect addressing.



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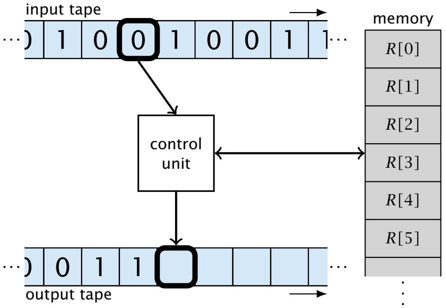

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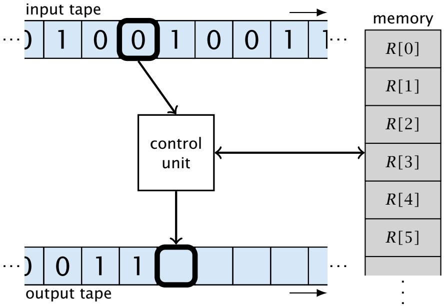

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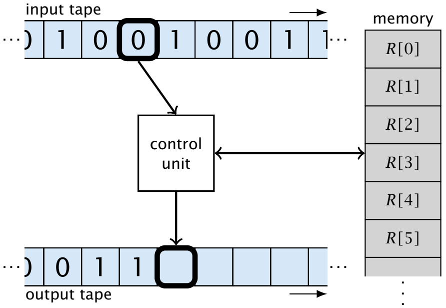

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Operations

- input operations (input tape → $R[i]$)
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- output operations ($R[i]$ → output tape)
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- register-register transfers
  - $R[j] := R[i]$
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- **indirect** addressing
  - $R[j] := R[R[i]]$
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    jumps to position x in the program;
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  - jumpz x $R[i]$
    jump to x if $R[i] = 0$
    if not the instruction counter is increased by 1;
  - jumpi i
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- uniform cost model
  Every operation takes time 1.
- logarithmic cost model
  The cost depends on the content of memory cells:
  - The time for a step is equal to the largest operand involved;
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Bounded word RAM model: cost is uniform but the largest value stored in a register may not exceed w, where usually $w = \log_2 n$.

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Example 2

Algorithm 1 RepeatedSquaring(n)

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r ← 2;
for i = 1 → n do
    r ← r^2
return r
```

running time

space requirement

4 Modelling Issues

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```

- running time:
  - uniform model: n steps
  - logarithmic model: $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1 = \Theta(2^n)$
- space requirement:
  - uniform model: $\mathcal{O}(1)$
  - logarithmic model: $\mathcal{O}(2^n)$

4 Modelling Issues

Example 2

Algorithm 1 RepeatedSquaring(n)

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1: r ← 2;
2: for i = 1 → n do
3:     r ← r^2
4: return r
```

- running time:
  - uniform model: n steps
  - logarithmic model: $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1 = \Theta(2^n)$
- space requirement:
  - uniform model: $\mathcal{O}(1)$
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There are different types of complexity bounds:

- best-case complexity:

$$C_{\text{bc}}(n) := \min\{C(x) \mid |x| = n\}$$

Usually easy to analyze, but not very meaningful.

- worst-case complexity:

$$C_{\text{wc}}(n) := \max\{C(x) \mid |x| = n\}$$

Usually moderately easy to analyze; sometimes too pessimistic.

- average case complexity:

$$C_{\text{avg}}(n) := \frac{1}{|I_n|} \sum_{|x|=n} C(x)$$

more general: probability measure μ

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