Lemma 12 (Chernoff Bounds)

Let $X_1,\dots,X_n$ be n independent 0-1 random variables, not necessarily identically distributed. Then for $X=\sum_{i=1}^n X_i$ and $\mu=E[X]$, $L\le\mu\le U$, and $\delta>0$

$$\Pr[X\ge(1+\delta)U]<\left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right)^{U},$$

and

$$\Pr[X\le(1-\delta)L]<\left(\frac{e^{-\delta}}{(1-\delta)^{1-\delta}}\right)^{L},$$

Lemma 13

For $0 \leq \delta \leq 1$ *we have that*

$$\left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right)^{U} \leq e^{-U\delta^2/3}$$

and

$$\left(\frac{e^{-\delta}}{(1-\delta)^{1-\delta}}\right)^{L} \leq e^{-L\delta^2/2}$$

Integer Multicommodity Flows

- Given s_i-t_i pairs in a graph.
- Connect each pair by a paths such that not too many path use any given edge.

$$\begin{array}{llrcl} \min & & W & & \\ \text{s.t.} & \forall i & \sum_{p \in \mathcal{P}_i} x_p & = & 1 \\ & & \sum_{p: e \in p} x_p & \le & W \\ & & x_p & \in & \{0,1\} \end{array}$$

Integer Multicommodity Flows

Randomized Rounding:

For each i choose one path from the set $\mathcal{P}_i$ at random according to the probability distribution given by the Linear Programming Solution.

Theorem 14

If $W^* \geq c \ln n$ *for some constant* c*, then with probability at least* $n^{-c/3}$ *the total number of paths using any edge is at most* $W^* + \sqrt{cW^* \ln n}$*.*

Integer Multicommodity Flows

Let X_e^i be a random variable that indicates whether the path for s_i-t_i uses edge e.

Then the number of paths using edge e is $Y_e = \sum_i X_e^i$.

$$E[Y_e] = \sum_i \sum_{p \in \mathcal{P}_i, e \in p} x_p^* = \sum_{p: e \in P} x_p^* \leq W^*$$

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Integer Multicommodity Flows

Choose $\delta = \sqrt{(c \ln n)/W^*}$.

Then

$$\Pr[Y_e \geq (1+\delta)W^*] < e^{-W^*\delta^2/3} = \frac{1}{n^{c/3}}$$

Integer Multicommodity Flows

Choose $\delta = \sqrt{(c \ln n)/W^*}$.

Then

$$\Pr[Y_e \geq (1+\delta)W^*] < e^{-W^*\delta^2/3} = \frac{1}{n^{c/3}}$$