
Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1 (10 Punkte)

n motorcyclists M_1, M_2, \dots, M_n start riding their bikes from a (straight) start line. At the start M_i and M_{i+1} are adjacent to each other. Each motorcyclist M_i starts at some angle ϕ_i and keeps riding in a straight line along this direction at a constant speed $s_i > 0$. Whenever a motorcyclist M_j comes across the path traversed by any other motorcyclist M_i , we say that M_i defeated M_j and in that case, M_j stops riding.

- We call the point where M_i defeats M_j as the point of ambush $A_{i,j} \in \mathbb{R}^2$. Show that if $A_{i',j'}$ is a point of ambush which occurs closest to the start line, then i' and j' are consecutive integers.
- Show how to enumerate in $O(n \log n)$ time, all events where one motorcyclist defeats another.

Aufgabe 2 (10 Punkte)

Give a sequence of m MAKESET, UNION and FIND operations, n of which are MAKESET operations, that take $\Omega(m \log n)$ time when we use union by rank only.

Aufgabe 3 (10 Punkte)

Show that a maximum flow in a network $G = (V, E)$ can always be found by a sequence of at most $|E|$ augmenting paths.

(*Hint:* Determine the paths after finding the maximum flow.)