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## Effiziente Algorithmen und Datenstrukturen I

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### Aufgabe 1 (10 Punkte)

Given the key of an element  $x$  in an  $n$ -node binary search tree (choose a BST with suitable properties) and a natural number  $i$ , show how to augment the tree to find the  $i$ -th successor of  $x$  in the linear order of the tree in  $O(\log n)$  time.

### Aufgabe 2 (10 Punkte)

Suppose that we wish to keep track of a *point of maximum overlap* in a set of intervals - a point that has the largest number of intervals in the set of intervals overlapping it.

1. Show that there will always be a point of maximum overlap which is an endpoint of one of the segments.
2. Design a data structure that efficiently supports the operations INSERT, DELETE, and FIND\_POM which are defined as follows:
  - (a) INSERT( $i, j$ ): Inserts the interval  $[i, j]$  in the set of intervals.
  - (b) DELETE( $i, j$ ): Deletes the interval  $[i, j]$  from the set of intervals.
  - (c) FIND\_POM: Returns a point of maximum overlap.

(*Hint:* Keep a red-black tree of all the endpoints. Associate a value of  $+1$  with each left endpoint, and associate a value of  $-1$  with each right endpoint. Augment each node of the tree with some extra information to maintain the point of maximum overlap.)

### Aufgabe 3 (10 Punkte)

Suggest how to use a skip list so that given a pointer to a node with key  $x$ , we can return a pointer to a node with key  $y < x$  in  $O(\log k)$  expected time where  $k$  is the distance between the nodes with values  $y$  and  $x$  in  $L_0$ .