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## Complexity Theory

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*Due date: June 26, 2012 before class!*

### Problem 1 (10 Points)

In the **SUCCINCT SET COVER** problem we are given a set  $S = \{\varphi_1, \dots, \varphi_m\}$  of 3DNF formulae on  $n$  variables, and an integer  $k$ . We need to decide if there exists a subset  $S' \subseteq \{1, \dots, m\}$  of size at most  $k$  such that  $\bigvee_{i \in S'} \varphi_i$  is a tautology.

- (i) Show that **SUCCINCT SET COVER**  $\in \Sigma_2^p$  by stating it as a  $\Sigma_2^p$  language.
- (ii) Since we know that  $\Sigma_2^p = \mathcal{NP}^{\mathcal{NP}}$ , give a formulation for **SUCCINCT SET COVER** that uses an  $\mathcal{NP}$  oracle.

### Problem 2 (10 Points)

Prove that the following language is **PSPACE**-complete:

**IN-PLACE ACCEPTANCE**: Given a Turing machine  $M$  and an input  $x$ , does  $M$  accept  $x$  without ever leaving the first  $|x| + 1$  symbols on its string?

### Problem 3 (10 Points)

Define the class  $\mathbf{E} = \bigcup_c \mathbf{DTIME}(2^{cn})$ .

- (i) Is  $\mathbf{E}$  closed under polynomial-time reductions?
- (ii) Show that  $\mathcal{P}^{\mathbf{E}} = \mathbf{EXP}$ .

### Problem 4 (10 Points)

Prove  $\mathbf{AL} = \mathcal{P}$ .