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Spring Semester 2012
Problem Set 6
June 12, 2012

Complexity Theory

Due date: June 19, 2012 **before** *class!*

Problem 1 (10 Points)

Prove the following:

(i) If for any $i \geq 1$, it holds that $\Sigma_i^p = \Pi_i^p$, then $\mathbf{PH} = \Sigma_i^p$.

(ii) If $\textsc{Sat} \preceq_m^p \overline{\textsc{Sat}}$, then $\mathbf{PH} = \mathcal{NP}$.

(iii) If there exists a **PH**-complete language, then the polynomial hierarchy collapses.

Problem 2 (10 Points)

(i) Argue that at least one of the assumptions $\mathbf{L} \neq \mathcal{P}$ and $\mathcal{P} \neq \mathbf{PSPACE}$ is true.

(ii) Use padding to show that if $\mathcal{P} = \mathbf{L}$, then $\mathbf{EXP} = \mathbf{PSPACE}$.

Problem 3 (10 Points)

Recall the definition of alternating Turing machines (ATM) with control states partitioned into sets $Q_\forall$ and $Q_\exists$, and the corresponding class **AP**.

(i) Show that a language $L \in \mathbf{AP}$ decided by an *existential* ATM (i.e. $Q_\forall = \emptyset$) is in $\mathcal{NP}$.

(ii) Show that a language $L \in \mathbf{AP}$ decided by an *universal* ATM (i.e. $Q_\exists = \emptyset$) is in co-$\mathcal{NP}$.

(iii) Show that $\mathbf{AP} = \text{co-}\mathbf{AP}$.

(iv) Show that **PSPACE** is contained in **AP** by showing that $\textsc{tQbf} \in \mathbf{AP}$.

Problem 4 (10 Points)

Suppose $t \geq 1, s > 0, t > s$. Then, $\mathbf{TISP}(n^t, n^s) \subseteq \Sigma_2\mathbf{TIME}(n^r)$ for any $r > \max\left(\frac{s+t}{2}, 1\right)$.
Hint: This is a generalization of a statement we used for proving $\textsc{Sat} \notin \mathbf{TISP}(n^{1.1}, n^{0.1})$. The proof is similar to the one presented in the lecture.