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Spring Semester 2012
Problem Set 5
June 5, 2012

Complexity Theory

Due date: June 12, 2012 **before** *class!*

Problem 1 (10 Points)

Define the class $\mathbf{DP} = \{L = L_1 \cap L_2 : L_1 \in \mathcal{NP}, L_2 \in \text{co-}\mathcal{NP}\}$. (Note that we do not know if $\mathbf{DP} = \mathcal{NP} \cap \text{co-}\mathcal{NP}$.) Consider the following languages:

$$\textsc{ExactIndset} = \{(G, k) : \text{ the largest independent set of } G \text{ has size exactly } k\},$$
$$\textsc{Critical Sat} = \{\varphi : \varphi \text{ is unsatisfiable, but deleting any clause makes it satisfiable}\}.$$

Show the following:

(i) $\textsc{ExactIndset} \in \Sigma_2^p = \mathcal{NP}^{\mathcal{NP}}$.

(ii) $\textsc{ExactIndset} \in \mathbf{DP}$.

(iii) $\textsc{Critical Sat}$ is $\mathbf{DP}$-complete.

Problem 2 (10 Points)

(i) Show that $2\textsc{Sat}$ is $\mathbf{NL}$-complete.

(ii) Show: If $A \preceq_m^{\log} B$, then $A \preceq_m^p B$.

Problem 3 (10 Points)

Give an example of a non-regular language that is in $\mathbf{SPACE}(\log\log)$.

Problem 4 (10 Points)

Consider the problem of checking a boolean formula's syntactical correctness. Show that this problem can be decided in log-space, even if we have no precedence relation between the boolean operators and force precedence behavior with parentheses, e.g. $(x \wedge y) \vee (\bar{z} \wedge x) \vee \bar{y} \vee z$ is a valid formula, as is $(x \wedge (y \vee \bar{z}) \wedge x) \vee \bar{y} \vee z$, while $x \wedge y \vee \bar{z} \wedge x \vee \bar{y} \vee z$ is not.