Technische Universität München
Fakultät für Informatik
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Spring Semester 2012
Problem Set 3
May 15, 2012

Complexity Theory

Due date: May 22, 2012 **before** *class!*

Problem 1 (10 Points)

Consider the problem of *map coloring*: Can you color a map with k different colors, such that no pair of adjacent countries has the same color?

(i) Describe the map coloring problem as a proper graph problem and redefine the language k-Colorability = {Maps that are colorable with at most k colors}.

(ii) Show that 2-Colorability is in $\mathcal{P}$.

(iii) Show that 3-Colorability is $\mathcal{NP}$-complete.
Hint: Use a reduction to Indset.

Problem 2 (10 Points)

Recall the following definition: A language A is *polynomial-time Cook-reducible* to a language B if there is a polynomial-time TM M that, given an oracle deciding B, can decide A.
Show that 3Sat is Cook-reducible to Tautology.

Problem 3 (10 Points)

In the Exactly One 3Sat problem, we are given a 3CNF formula φ and need to decide if there exists a satisfying assignment u for φ such that every clause of φ has exactly one true literal. Prove that Exactly One 3Sat is $\mathcal{NP}$-complete.

Problem 4 (10 Points)

Suppose $L_1, L_2 \in \mathcal{NP} \cap \text{co-}\mathcal{NP}$. Then show that $L_1 \oplus L_2$ is in $\mathcal{NP} \cap \text{co-}\mathcal{NP}$, where $L_1 \oplus L_2 = \{x : x \text{ is in exactly one of } L_1, L_2\}$, i.e. an XOR.