6.5 Transformation of the Recurrence

Example 9

$$\begin{aligned} f_0 &= 1 \\ f_1 &= 2 \\ f_n &= f_{n-1} \cdot f_{n-2} \text{ for } n \geq 2\ . \end{aligned}$$

Define

$$g_n := \log f_n\ .$$

Then

$$\begin{aligned} g_n &= g_{n-1} + g_{n-2} \text{ for } n \geq 2 \\ g_1 &= \log 2 = 1,\ g_0 = 0 \ (\text{fÃČÅŠr } \log = \log_2) \\ g_n &= F_n \ (n\text{-th Fibonacci number}) \\ f_n &= 2^{F_n} \end{aligned}$$

6.5 Transformation of the Recurrence

Example 10

$$f_1 = 1$$
$$f_n = 3f_{\frac{n}{2}} + n;\ \text{for } n = 2^k\ ;$$

Define

$$g_k := f_{2^k}\ .$$

6.5 Transformation of the Recurrence

Example 10

Then:

$$\begin{aligned} g_0 &= 1 \\ g_k &= 3g_{k-1} + 2^k,\ k \geq 1 \end{aligned}$$

We get,

$$\begin{aligned} g_k &= 3^{k+1} - 2^{k+1},\ \text{hence} \\ f_n &= 3 \cdot 3^k - 2 \cdot 2^k \\ &= 3(2^{\log 3})^k - 2 \cdot 2^k \\ &= 3(2^k)^{\log 3} - 2 \cdot 2^k \\ &= 3n^{\log 3} - 2n\,. \end{aligned}$$