Example: $a_n = 3a_{n-1} + n, a_0 = 1$

5. Write $f(z)$ as a formal power series:

$$\begin{aligned} A(z) &= \frac{7}{4}\cdot\frac{1}{1-3z} - \frac{1}{4}\cdot\frac{1}{1-z} - \frac{1}{2}\cdot\frac{1}{(1-z)^2} \\ &= \frac{7}{4}\cdot\sum_{n\ge 0}3^n z^n - \frac{1}{4}\cdot\sum_{n\ge 0} z^n - \frac{1}{2}\cdot\sum_{n\ge 0}(n+1)z^n \\ &= \sum_{n\ge 0}\Big(\frac{7}{4}\cdot 3^n - \frac{1}{4} - \frac{1}{2}(n+1)\Big)z^n \end{aligned}$$

6. This means $a_n = \frac{7}{4}3^n - \frac{1}{2}n - \frac{3}{4}$.

6.5 Transformation of the Recurrence

Example 9

$$\begin{aligned} f_0 &= 1 \\ f_1 &= 2 \\ f_n &= f_{n-1}\cdot f_{n-2} \text{ for } n \ge 2 . \end{aligned}$$

Define

$$g_n := \log f_n .$$

Then

$$\begin{aligned} g_n &= g_{n-1} + g_{n-2} \text{ for } n \ge 2 \\ g_1 &= \log 2 = 1,\ g_0 = 0 \text{ (fÃČÅŠr } \log = \log_2) \\ g_n &= F_n \text{ (n-th Fibonacci number)} \\ f_n &= 2^{F_n} \end{aligned}$$

6.5 Transformation of the Recurrence

Example 10

$$\begin{aligned} f_1 &= 1 \\ f_n &= 3f_{\frac{n}{2}} + n; \text{ for } n = 2^k ; \end{aligned}$$

Define

$$g_k := f_{2^k} .$$

6.5 Transformation of the Recurrence

Example 10

Then:

$$\begin{aligned} g_0 &= 1 \\ g_k &= 3g_{k-1} + 2^k,\ k \ge 1 \end{aligned}$$

We get,

$$\begin{aligned} g_k &= 3^{k+1} - 2^{k+1}, \text{ hence} \\ f_n &= 3\cdot 3^k - 2\cdot 2^k \\ &= 3(2^{\log 3})^k - 2\cdot 2^k \\ &= 3(2^k)^{\log 3} - 2\cdot 2^k \\ &= 3n^{\log 3} - 2n . \end{aligned}$$