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Randomisierte Algorithmen

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Aufgabe 1

The algorithm for approximating the number of connected components asks one to choose a Random Variable that satisfies $Pr[X \geq k] = \frac{1}{k}$ for $k \geq 1$. How is this Random Variable chosen? That is, come up with an appropriate random experiment and show that it is correct.

Aufgabe 2

The original approximation algorithm for MST cost is as follows:

ApproxMST(G,n,r){
  for($i = 1$ to $W - 1$)
    $\hat{c}(i)$ = ApproxConnectedComps($G(i), r$)
  return $n - W + \sum_{i=1}^{W-1} \hat{c}(i)$
}

ApproxConnectedComps($G(i), r$){
  for($j = 1$ to r){
    Uniformly and randomly choose v from V.
    Randomly choose X to satisfy the distribution $Pr[X \geq k] = \frac{1}{k}$
    Begin a BFS from v until...
     (a) X nodes are visited
     (b) All nodes in the connected component of v are visited
    In case (a), $b[j] = 0$; in case (b), $b[j] = 1$.
  }
  return $\frac{n}{r} \sum_{j=1}^{r} b[j]$
}

The runtime for this algorithm is $O(WD \log \frac{n}{\rho\epsilon^2})$
Now consider the following:

ApproxMST(G,n,r){
  $\hat{c}$ = ApproxConnectedComps(G, r)
  return $n - W + \hat{c}$
}

ApproxConnectedComps(G, r){

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for(j = 1 to r){
    Uniformly and randomly choose v from V.
    Randomly choose X to satisfy the distribution Pr[X ≥ k] = 1/k
    Compute largest j* s.t. X ≥ |c(j*)|
    ĉ(j) = j* · n
}
return (1/r) Σ_{j=1}^{r} ĉ(j)
}
```

The runtime for this algorithm has been improved to $O(D \log \frac{n}{\rho\epsilon^2})$. Is it still correct? Explain your answer.