$n=2^k \qquad \vec{a}=(a_0, a_1, a_2, \dots\ a_{n-1})$

$$P_{\vec{a}}(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + \dots$$

$$\vec{a}_g = (a_0, a_2, \dots, a_{n/2}-1)$$

$$P_{\vec{a}_g}(x^2) = a_0x^{2\cdot 0} + a_2x^{2\cdot 1} + a_4x^{2\cdot 2} + \dots\ a_{n/2-1}x^{\frac{n}{2}-1}$$

$$\vec{a}_u = (a_1, a_3, a_5, \dots, a_{n/2})$$

$$P_{\vec{a}_u}(x^2) = a_1x^{2\cdot 0} + a_3x^{2\cdot 1} + a_5x^{2\cdot 2} + \dots\ a_{n/2}x^{n/2}$$

$$x \cdot P_{\vec{a}_u} = a_1x + a_3x^3 + a_5x^5 + \dots$$

$$\underset{\omega^j}{P_{\vec{a}}(x)} = P_{\vec{a}_g}(\overset{y}{\underset{\omega^{2j}}{x^2}}) + \underset{\omega}{x} \cdot P_{\vec{a}_u}(\overset{y}{\underset{\omega^{2j}}{x^2}}) \quad : j = 0, 1, 2, \dots$$

$$F_{n/2-1,\,\omega^2}(\vec{a}_g) = (\underset{c_0}{P_{\vec{a}_g}(1)},\ \underset{c_1}{P_{\vec{a}_g}(\omega^2)},\ \dots\ )$$

$$(\omega)F_{n/2,\,\omega^2}(\vec{a}_u) = (1\cdot \underset{d_0}{P_{\vec{a}_u}(1)},\ \omega\, \underset{d_1}{P_{\vec{a}_u}(\omega^2)},\ \dots)$$

$e_0 = c_0 + 1\cdot d_0$

$e_1 = c_1 + \omega d_1$

$\vdots$

$$(\omega^2)^{n/2} = \omega^n = 1$$

Beh: $\sum_{j=0}^{n-1} \omega^{kj} = 0$

für $k = 1, 2, \ldots, n-1$

Zwischenbehauptung

$$\sum_{j=0}^{n-1} a^j = \frac{a^n - 1}{a - 1} \quad , a \in \mathbb{C}$$

Bew: Vollst. Ind. über n.

$$n = 1: \quad \sum_{j=0}^{0} a^0 = 1 = \frac{a^1 - 1}{a - 1}$$

$$n > 1: \quad \sum_{j=0}^{n-1} a^j = \sum_{j=0}^{(n-1)-1} a^j + a^{n-1} \underset{\text{I.V.}}{=}$$

$$\frac{a^{n-1} - 1}{a - 1} + a^{n-1} = \frac{a^{n-1} - 1 + (a-1)a^{n-1}}{a-1}$$

$$= \frac{a^{n-1} - 1 + a^n - a^{n-1}}{a-1} = \frac{a^n - 1}{a - 1}.$$