Excitable Media

MB - JASS 09

Outline

- Theorie of excitable media Chris
  - Describing excitable media:
    - Cellular automata (Prof Hensel-algorithm, deactivating, activating)
    - Differential equations(Brüsselator etc.)
- Real excitable media (Belousov-Zhabotinsky-reagents) Daniel
  - Chemical activation/deactivation of Ferroin due to BZ-reaction
- Experiments Chris
  - Simulation on PC
  - Demonstration (effect of disturbance→Rotor)
  - Demo on
- What is a Rotor?Daniel
  - origin
    - phase-singularity,
    - rubber clock-hand
  - Properties:
    - Open ends
    - Radius of curvature

Excitable Media

- Definition:
  - Dynamic system of elementary cells
  - Continuously distributed in space
  - Interaction processes:
    - Local coupliong(reaction)
    - Diffusion-like transport processes

- Examples
  - Action Potential on nerve fibers
  - Forest fire
  - Belousov-Zaboutinsky reagents

  → Competing of activation and deactivation of media

Describing Excitable Media

- Time Course
  - Activator(A) /Deactivator(B)

$$\frac{dA(x,y,t)}{dt} = f_{react}(A(x,y,t), B(x,y,t)) + f_{diff}(A(x,y,t), B(x,y,t))$$

- Diffusion
  - First law of Fick (J:flux;c:concentration)

$$J = -D\frac{\partial c}{\partial x} = -D\vec{\nabla} \bullet \vec{c}$$

  - Continuity equation

$$\frac{\partial \vec{c}}{\partial t} = -\vec{\nabla} J$$

  - Second law of Fick (D: diffusion coefficient)

$$\frac{\partial \vec{c}}{\partial t} = -\vec{\nabla}(D\vec{\nabla} \bullet \vec{c}) = -D\Delta\vec{c}$$

Describing Excitable Media

- Reaction Models
  - Fitz-Nagumo Model
    - Spatial-fixed Inhibitor(potential)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f(u) - V$$

$$\frac{\partial V}{\partial t} = \varepsilon(\gamma u - V)$$

  - Brusselator
    - Chemical reaction system

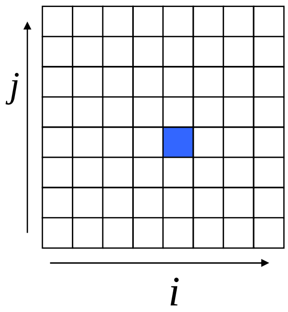
$$\frac{dX}{dt} = k_1 A - k_2 BX + k_3 X^2 Y - k_4 X$$

$$\frac{dY}{dt} = k_2 BX - k_3 X^2 Y$$

I	A	→	X
II	B + X	→	Y + C
III	2X + Y	→	3X
IV	X	→	D
V	A + B	→	C + D

Simulating excitable media

- Cellular Automata



$$dA_{ij} = (f_{react}(A_{ij}, B_{ij}) + f_{diff}(A_{?,?})) \times dt$$

Discrete Diffusion

$$\frac{d^2 A}{dx^2} + \frac{d^2 A}{dy^2} = \frac{A_{i+1,j} + A_{i-1,j} + A_{i,j+1} + A_{i,j-1} - 4A_{i,j}}{w^2}$$

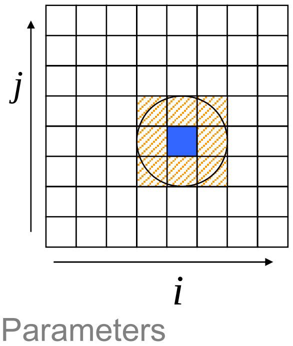
- Dummy Code
  - For i in columns
  - For j in rows
    - A_{ij}= A_{ij}+diffuse(A_{ij})+react(A_{ij})

Simulating excitable media

- Problem:
  - Stability vs. Precession
- Solving coupled linear equations for diffusion
  - Euler backward
  - Crank-Nicholson
  - …
- Solving non-linear equations for reaction
  - Is there an easy solution???

Simulating excitable media

- Excitation rule of CA



$$\Psi = \sum_{\Delta i^2 + \Delta j^2 <= r^2} A_{i+\Delta i, j+\Delta j} * \mathrm{ker}\,nel(\Delta i, \Delta j)$$

$$A_{i,j}(t+dt) = \begin{cases} A_{\max} : \Psi \geq \alpha \\ A_{i,j}(t) - \beta : \Psi < \alpha \end{cases}$$

- Parameters
  - Alpha: excitation-treshold
  - Beta: max/beta = lifetime of excitation
  - Max: Maximal excitation

Simulating excitable media

- Let‘s simulate an excitable media:
  - Kernel gaussian
  - Showing effects of alpha and beta and max
  - Show linkzones
- Let‘s visit the real world:
  - Belousov-Zhaboutinsky-reagents

Belousov-Zhabotinsky-Reaktion

- Nonlinear temporal chemical oscillator
- To viewable the status of the reaction equilibrium a redox indicator is required (here: ferroin)
- Chemical equation of the BZ-reaction

I $2\ Br^- + BrO_3^- + 3\ H_+ + 3\ H_2Mal \rightarrow 3\ HBrMal + 3\ H_2O$

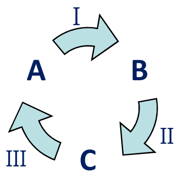
II $BrO_3^- + 4\ Ferroin^{2+} + H_2Mal + 5\ H^+ \rightarrow 4\ Feriin^{3+} + HBrMal + 3\ H_2O$

III $4\ Feriin^{3+} + HBrMal + 2\ H_2O \rightarrow 4\ Ferroin^{2+} + HCOOH + 2CO_2 -+ 5\ H^+ + Br-$

IV $3\ BrO3- + 5\ "H2Mal" + 3\ H+ \rightarrow 3\ HBrMal + 2\ HCOOH + 4\ CO_2 + 5\ H_2O$

Scheme for the reaction process

Simplified exemplar:



- **Bromate convertes to molecular bromine → attacks malonic acid and brominates it**
- **Bromate is decomposed**
- **At high bromide, a series of high oxygenatom transfer is involve → bromide is consumed**

Chemical Oscillation

- Bromide below a critical concentration → free radical mechanism (II) dominates
- Bromide below this threshold → $HBrO_2$ appears explosively by a reaction, which consumes the last bromide
- Red ferrous ferroin oxidizes to the blue ferric
- It Turns back to the red form and bromides
- Enough bromide → first reaktion

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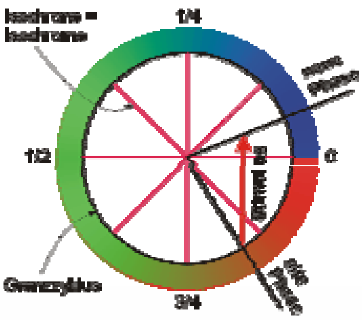
The BZ-reaction in real

- Start BZ-reaction
  - (picture maybe here)

- Lift the petri-disc→ effect of perturbation
- Can we see the effect in the simulation?
- Show simulation with perturbation
- What happend?
- Now explanation with rubber-clock hand phase singularity

Phase singularity

- Simple example for the limit cycle oscillator is the rubber clock-hand
- Interruption of the continuous phase due to a exterior stimulus and after that a new phase is beginning
- The stimulus misplaced the clock, whereby the clock is fast or slow



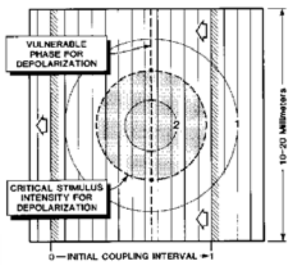
Critical stimulation

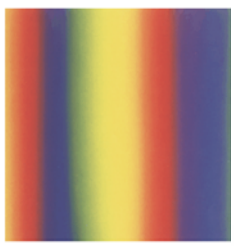
- Critical stimulation → rubber clock hand gets in the middle of the clock, whereby the clock loses his proper time
- The stimulation is in a phase singularity, which position is defined by a critical phase and a critical amplitude

Animation rubber clock hand (start.exe)

Consequences for excitable media

- Excitable media is a coupled limit cycle oscillators (composed of many cells, like the myocardium)
- Stimulation by an amplitude, that is bigger than the critical stimulation → critical combination by coupling interval and intensity of the stimulus
- Both points rest on the cutting side of the critical phase and the critical intensity of the stimulus





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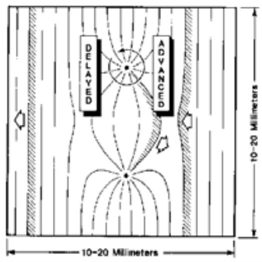
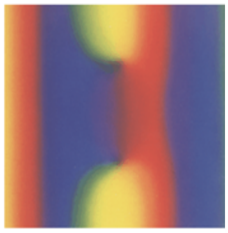
Rotor

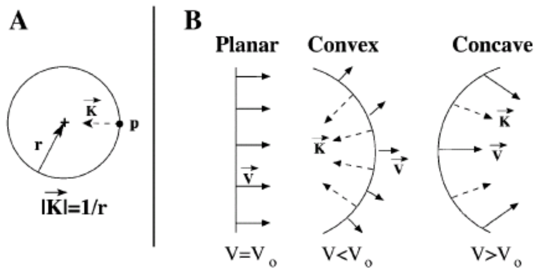
- On both sides, where the critical combination of old phase and stimulation-amplitude is reached, rotors are developping
- Rotors have spirally waves fragments with open ends, rotating around both phase singularities
- Rotors can't stopped by interaction with regular waves
- They also occur on human heart (atrial or ventricular fibrillation / sudden cardiac death)





Radius of curvature

- Important functional mechanism of cardiac arrhythmias
- Propagation of a { convex concave } wavefront is { slower faster } than of a planar wavefront
- Front of the wave distributed over a larger area of unexcited media ahead of the wave (convex)



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Speed of a curvature wave

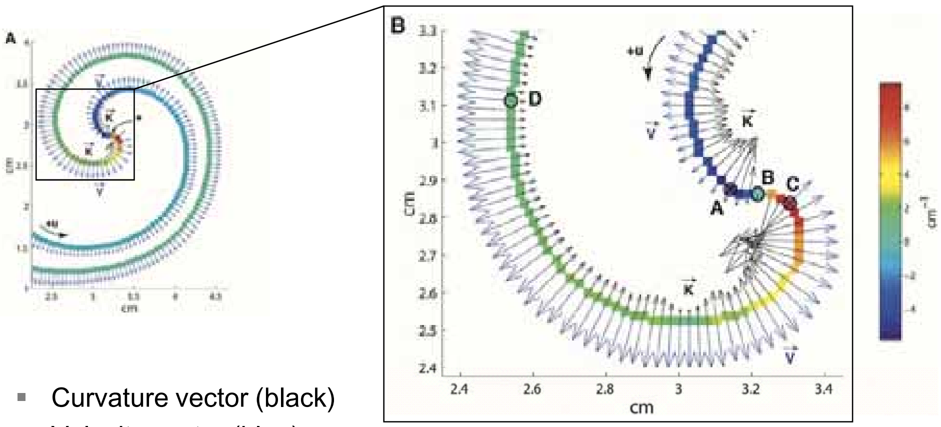
- Curvature of a wavefront: K
- Propagation speed of a wavefront: V
- Speed of a planar wave: V0
- Diffusion coefficient of the medium: D

$$V(K) = V_0 - DK$$

- Consequence: propagation will not occur when the curvature of the wavefront is above a critical level

$$K_{cr} = \frac{V_0}{D} \Rightarrow V(K_{cr}) = 0$$

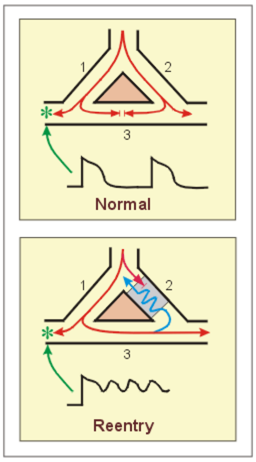
Critical radius



- Curvature vector (black)
- Velocity vector (blue)
- Asterisk point: wavefront meets waveback; curvature and velocity vectors flip to the opposite side of the isopotential

Rotors on the heart

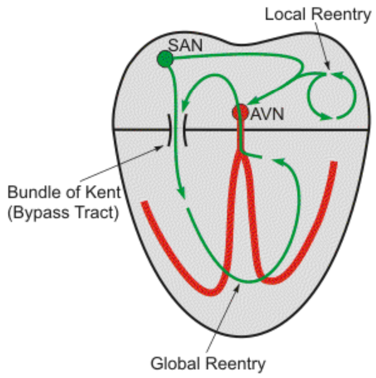
- Reentry
  - local
    - Atrium, ventricle
    - Purkinje fibers
  - 3 evil incredients
    - Unidirectional conduction block
    - Right timing
    - Refractory period of tissue
  - Abolishing reentry
    - Conduction speed
    - Refractory period
    - Drugs: beta-blockers, ACE-inhibitors



- SHOW SIMULATION of reentry

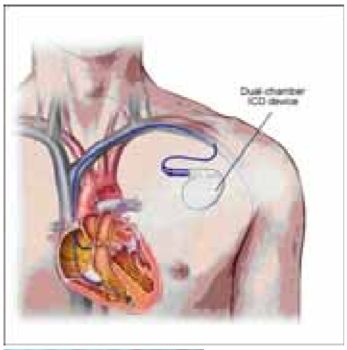
Rotors on the heart

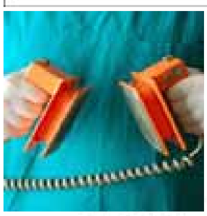
- Reentry
  - global
    - Wolf-White-Syndrom
    - AV-reentry



How to stop reentries?

- Drugs prevent occurrence
  - Antiarrythmics
    - Beta-blockers
    - ACE-inhibitors
- Defibrillation
  - Extern
  - ICD





Background to Defibrillation

- Show simulation!!!

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