Dynamic inverse problem in acoustic media

Denis Anikiev

St.Petersburg State University

Faculty of Physics, Earth Physics Department

Laboratory of Elastic Media Dynamics (supervisor: Dr. Boris Kashtan)

JASS 2009

March 29-April 7 2009 St.Petersburg, Russia

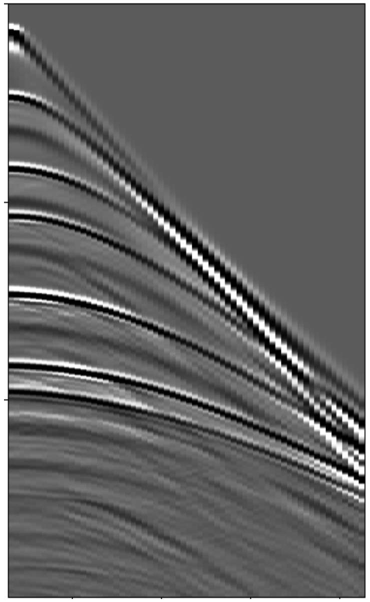
Euler International Mathematical Institute

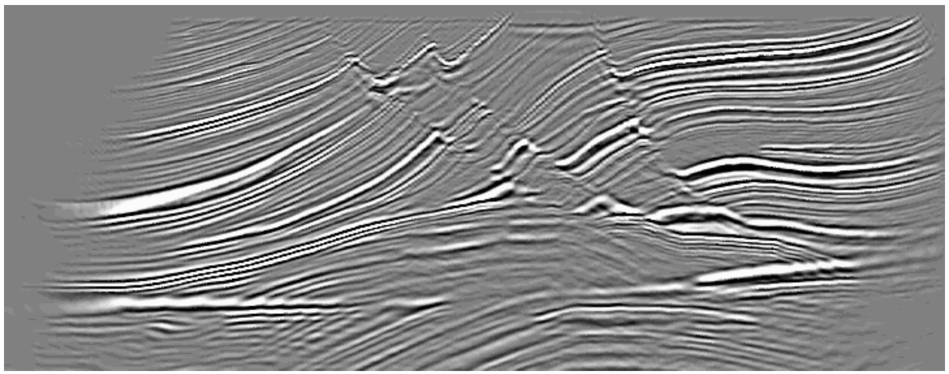
Outline

- Motivation
- Main principles of the method
- Numerical examples
- Conclusions
- Future work

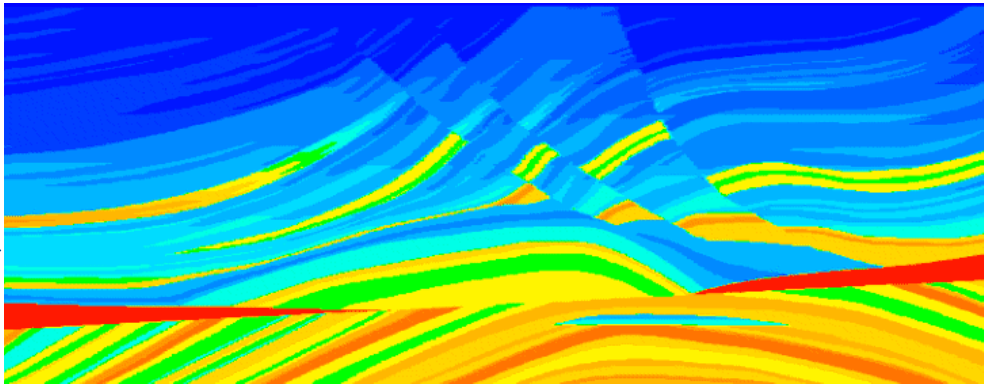
Motivation

Seismic data → Processing → INVERSION → Reconstructed velocity





INTERPRETATION →



Motivation

Kirchhoff migration – good for reconstructing of horizons

Full wave tomography – good for absolute velocity values

BUT: both methods need a proper starting model

How we can obtain a good starting model?

Method

There exist situations when we can get the unique solution

For instance, in case of acoustic equation:

$$\frac{1}{\rho c^2} U_{tt} - \frac{\partial}{\partial z}\left(\frac{1}{\rho} U_z\right) - \frac{\partial}{\partial x}\left(\frac{1}{\rho} U_x\right) = 0$$

Initial conditions:

$$U \;|_{t<0} = 0$$

Boundary conditions (source):

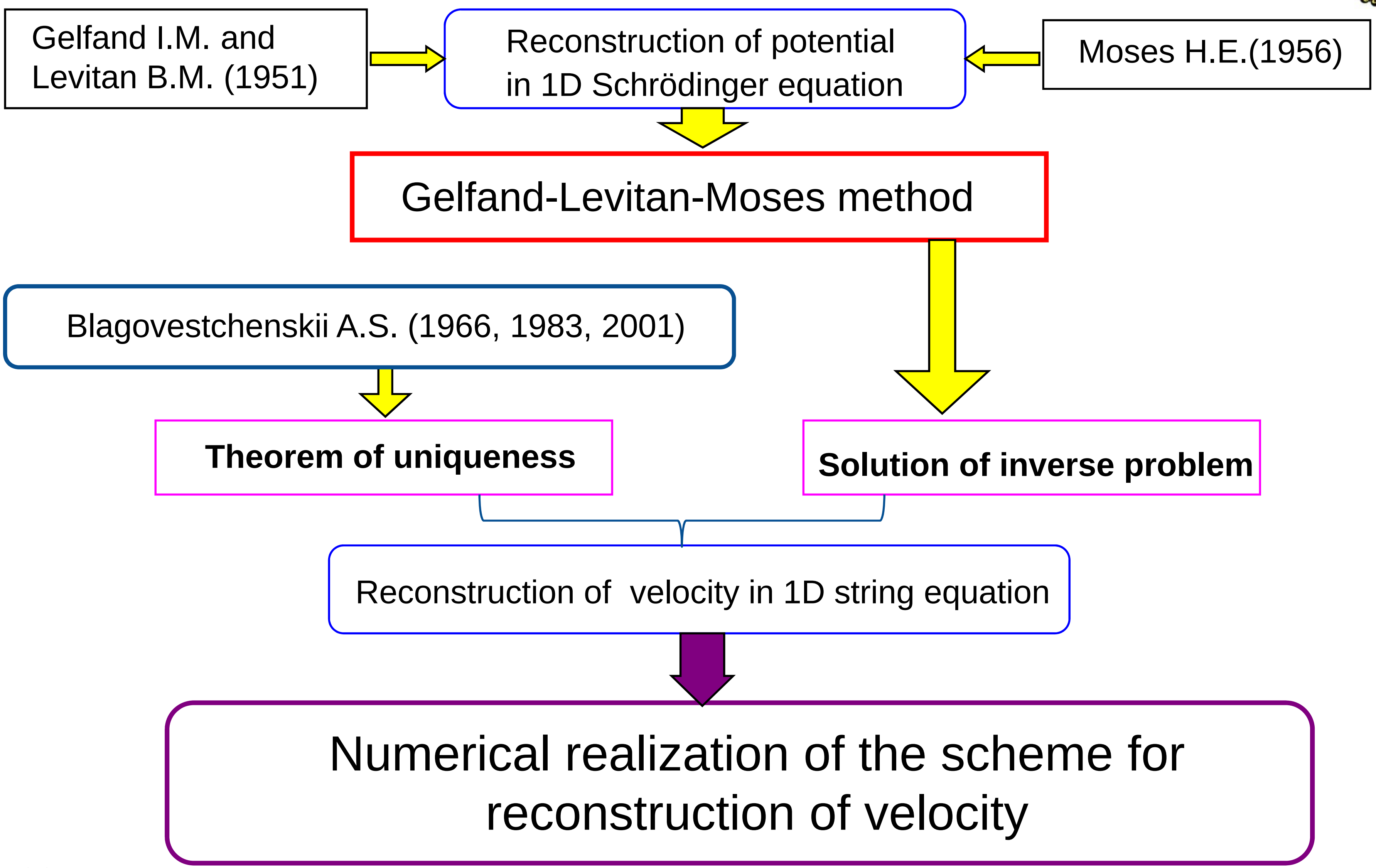
$$U \;|_{z=0} = \delta(t)\delta(x)$$

Observations on free surface:

$$U_z \;|_{z=0} = R(t, x)$$

$R(t,x)$ ⇒ $c(x,z), \rho(x,z)$ Velocity and density

Brief history of GLM method



Method

$$\rho = 1$$

$$U_{tt} - c^2(z)U_{zz} = 0$$
$$U\,|_{t<0} = 0$$
$$U\,|_{z=0} = f(t)$$

MEASURED

$$U_z\,|_{z=0} = R(t)$$

$$\int K(R(t-s))\,v(s)\,ds = S(R(t-s))$$

$$v$$

$$c(z)$$

$$f(t) \neq \delta(t)$$

~~theorem of uniqueness~~

$$f(t) = \delta(t)$$

Global theorem of uniqueness

Method

$$f(t) = \delta(t)$$

$$R_1(t) = R(t) - \delta'(t),$$

$R_1^+(t)$ is odd continuation of function $R_1(t)$ to negative time

Gelfand-Levitan-Moses equation of second kind

$$v(t,y) - \frac{1}{2}\int_{-y}^{y} d\tau\, v(\tau,y)\int_{\tau}^{y} d\eta R_1^+(t-\eta) = \frac{1}{2}\int_{-y}^{y} d\tau R_1^+(t-\tau)$$

$$v(t,y)$$

$$q(y) = -2v_y(y,y)$$

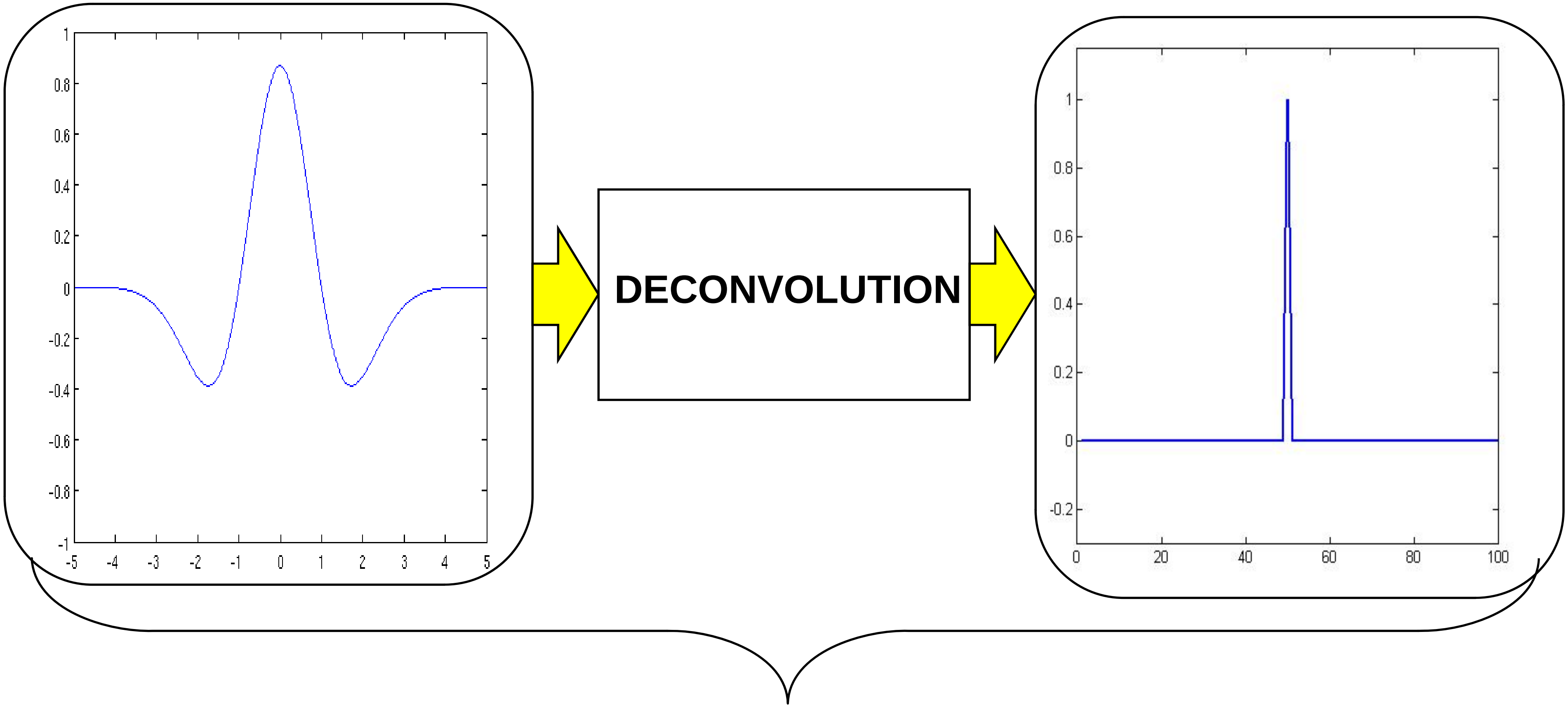
$$\sigma(y): \quad q(y) = \frac{(\sqrt{\sigma})''}{\sqrt{\sigma}} + k_x^{\,2}\frac{1}{\sigma^2}$$

$$k_x = 0$$

$$c(y) = \frac{1}{\sigma}$$

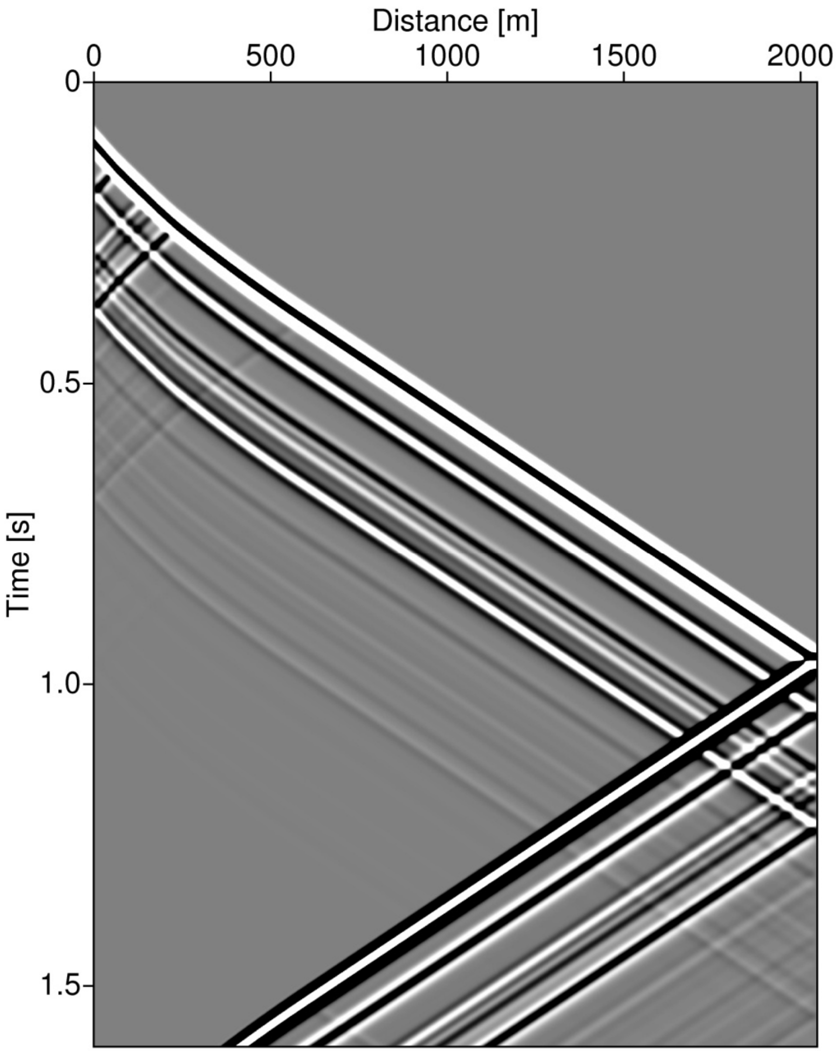
$$c(z): \quad z = \int_0^y c(y)dy$$

How to obtain delta function?

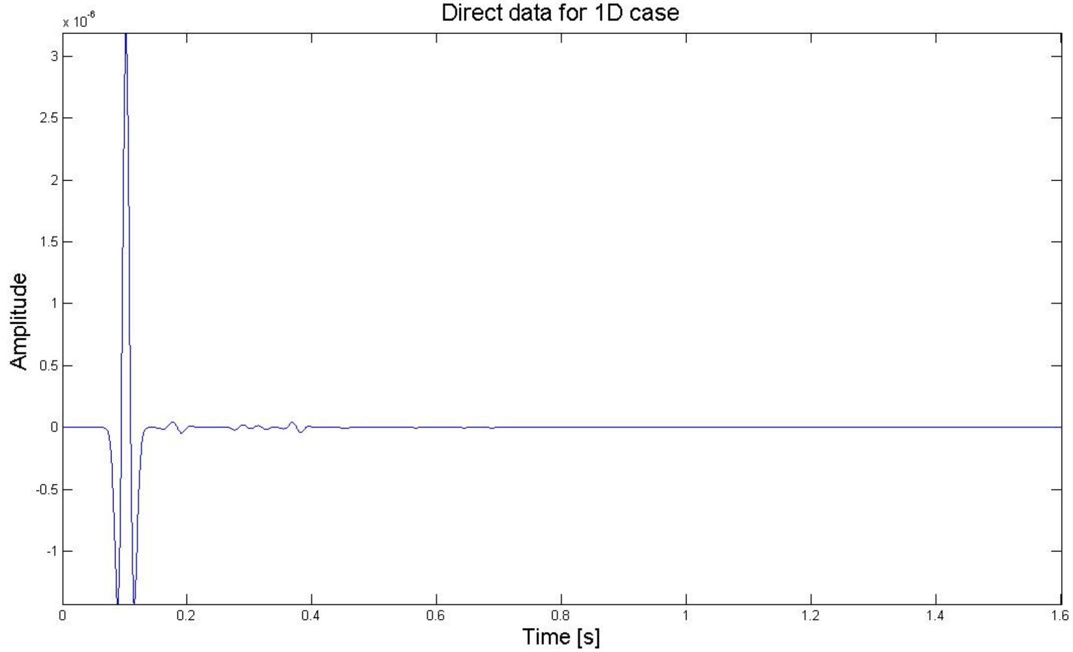


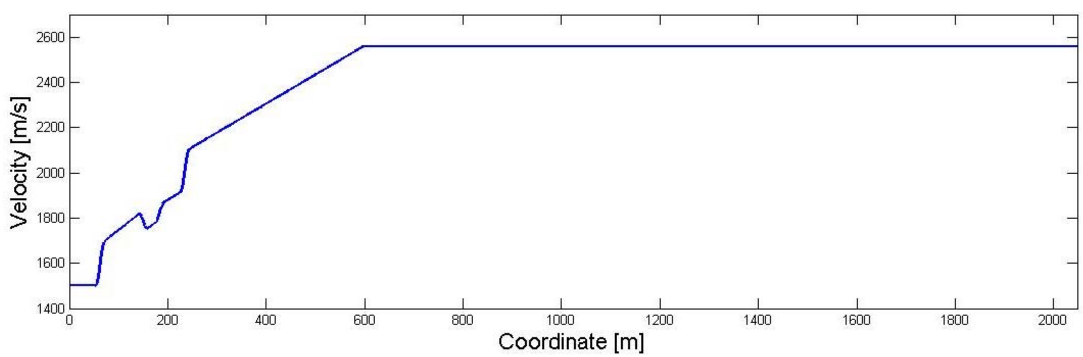
Wiener filter for the Ricker wavelet

Numerical result for 1D

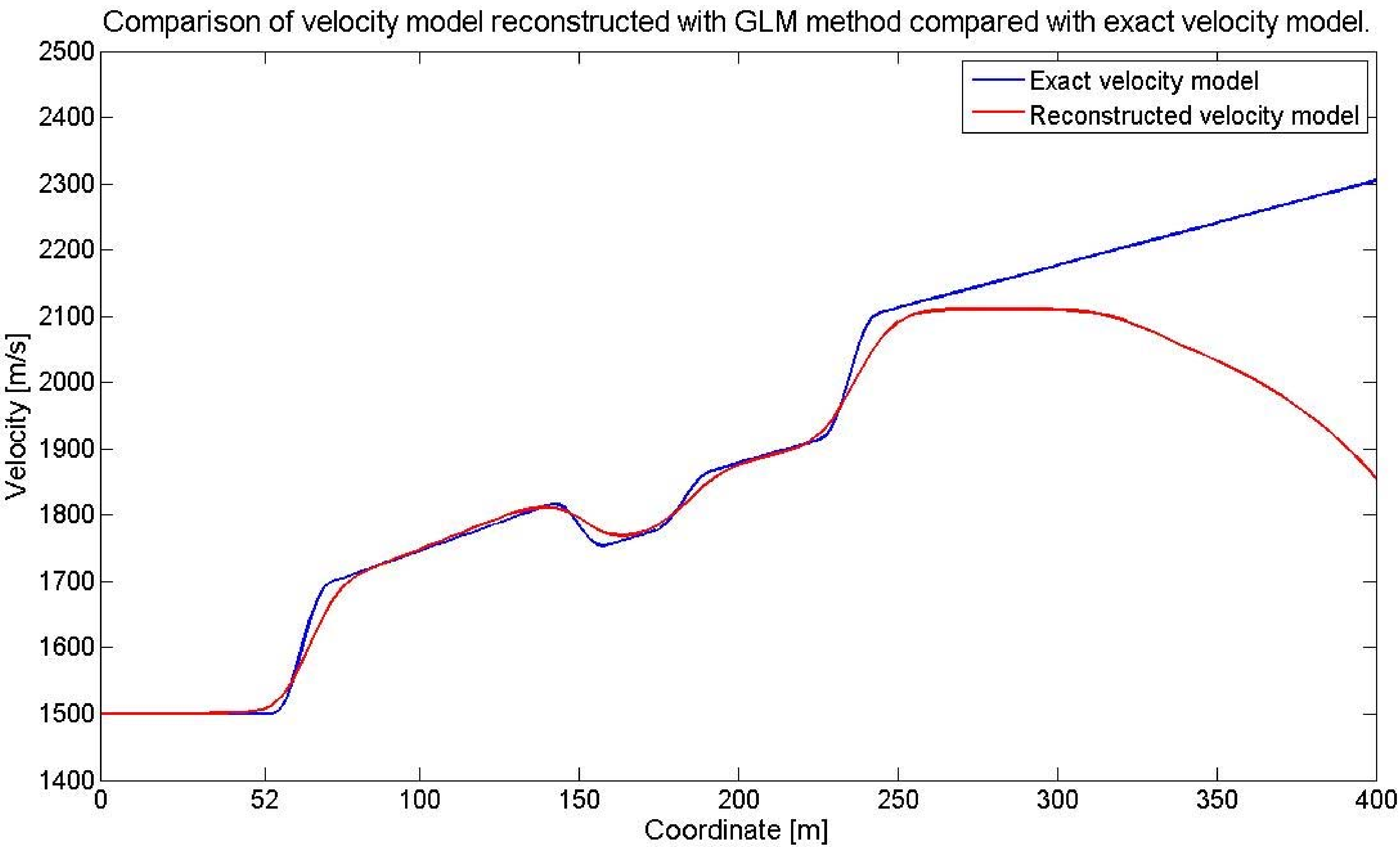


seismogram section of 1-D acoustic modelling

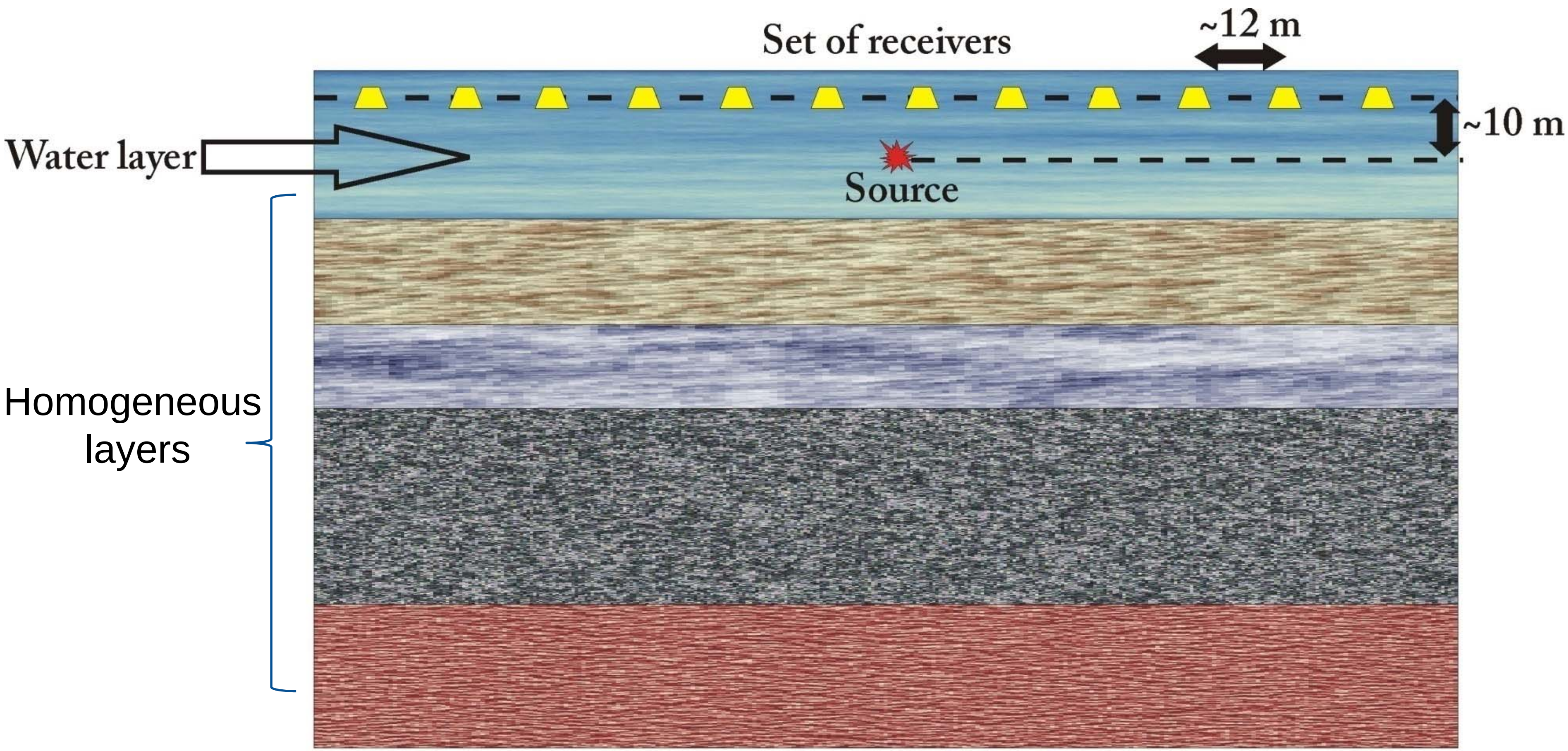




Numerical result for 1D



Standard scheme of acquisition for 2D



Fourier transform for space coordinate in 2D case

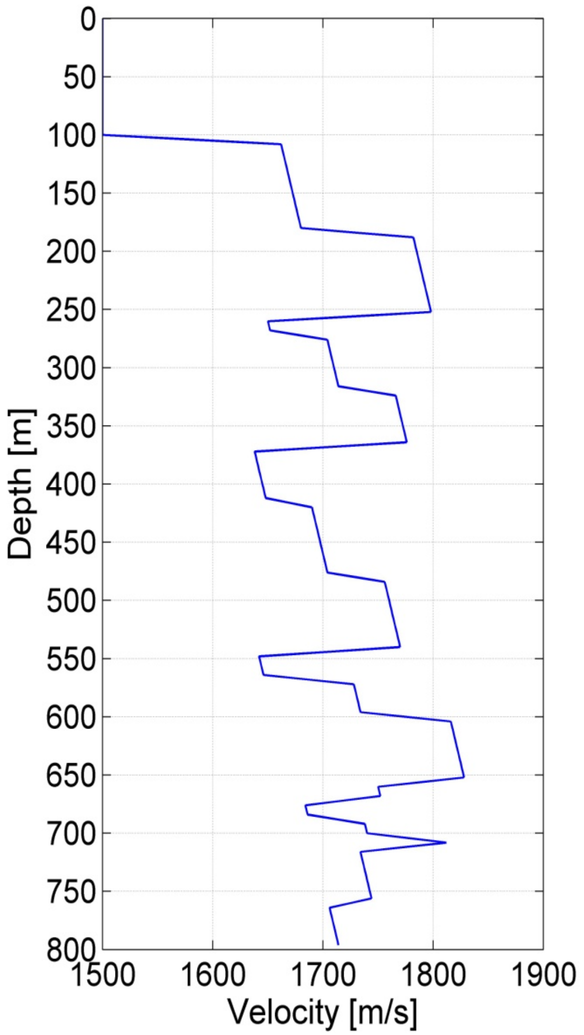
$$R(t,k_x) = \int R(t,x)\cos(k_x x)dx \approx \triangle x \sum_{n=1}^{N} R(t,x_n)\cos(k_x x) =$$

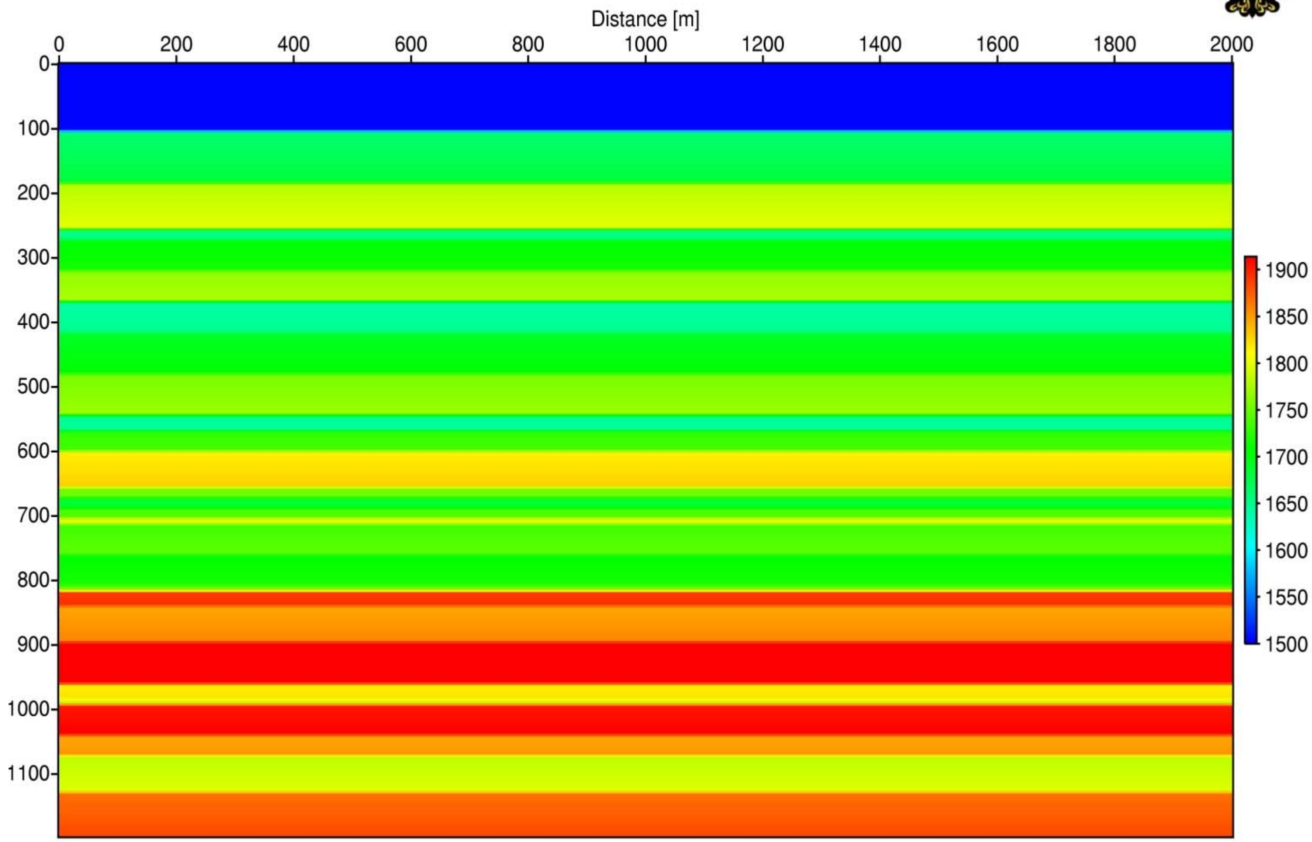
$$if \qquad k_x = 0$$

$$= \triangle x \sum_{n=1}^{N} R(t,x_n)$$

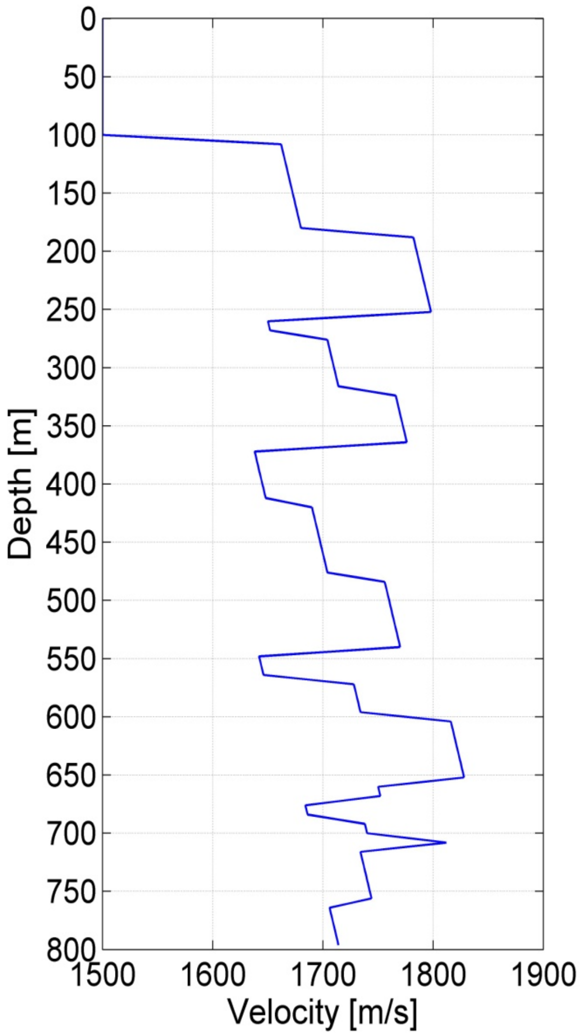
From 2D to 1D via Fourier transform

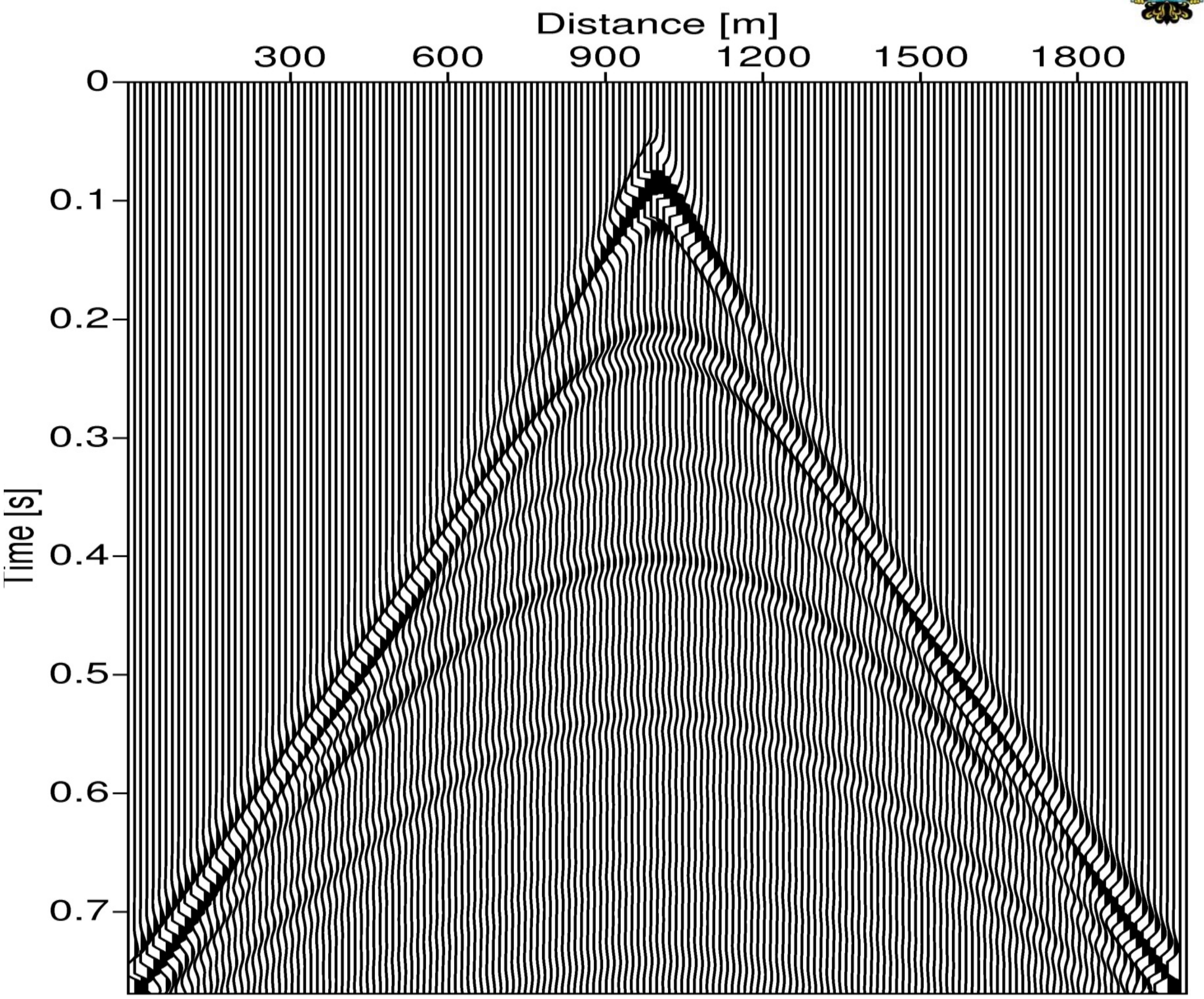
Numerical results for 2D



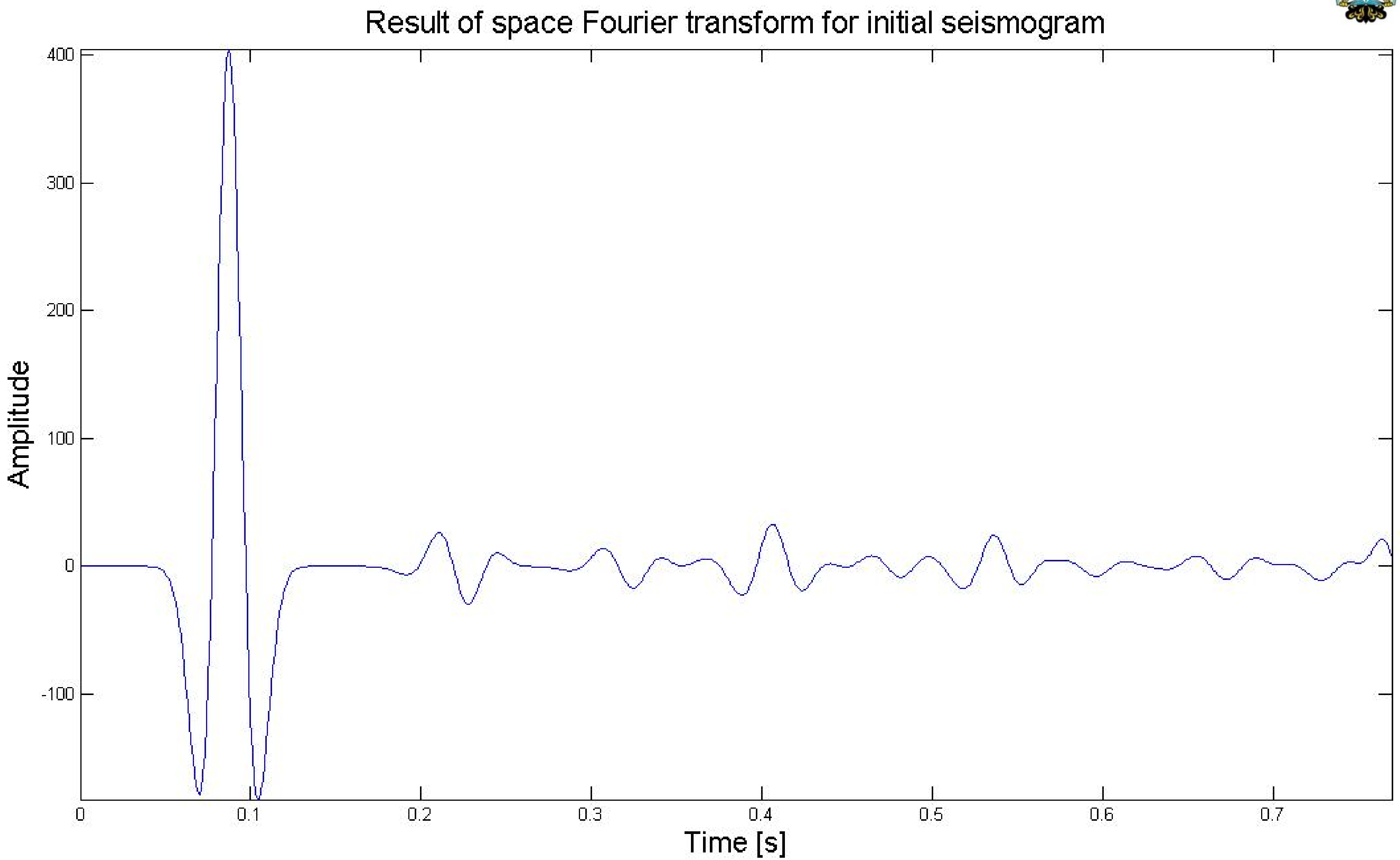


Numerical results for 2D

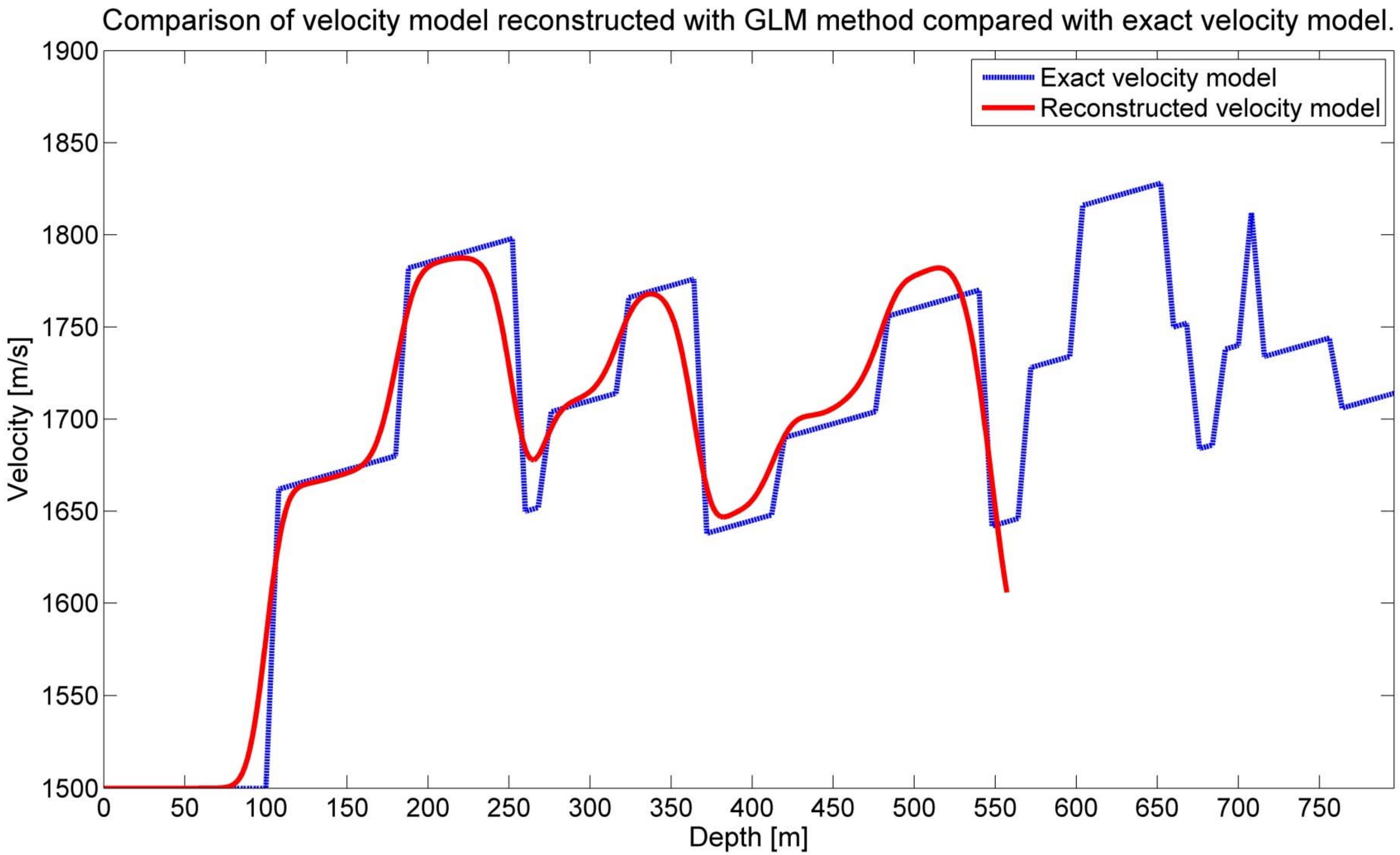




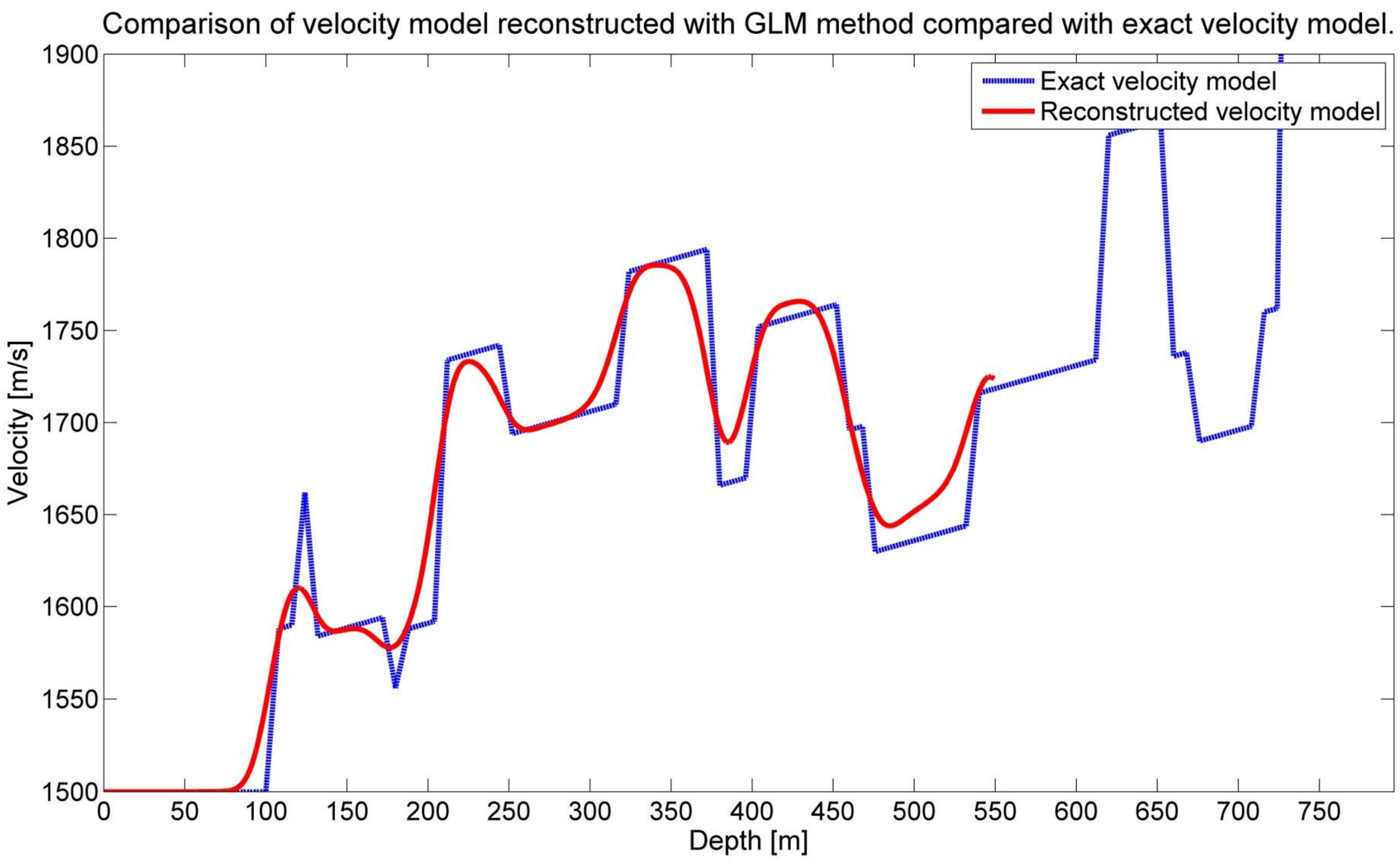
Numerical results for 2D



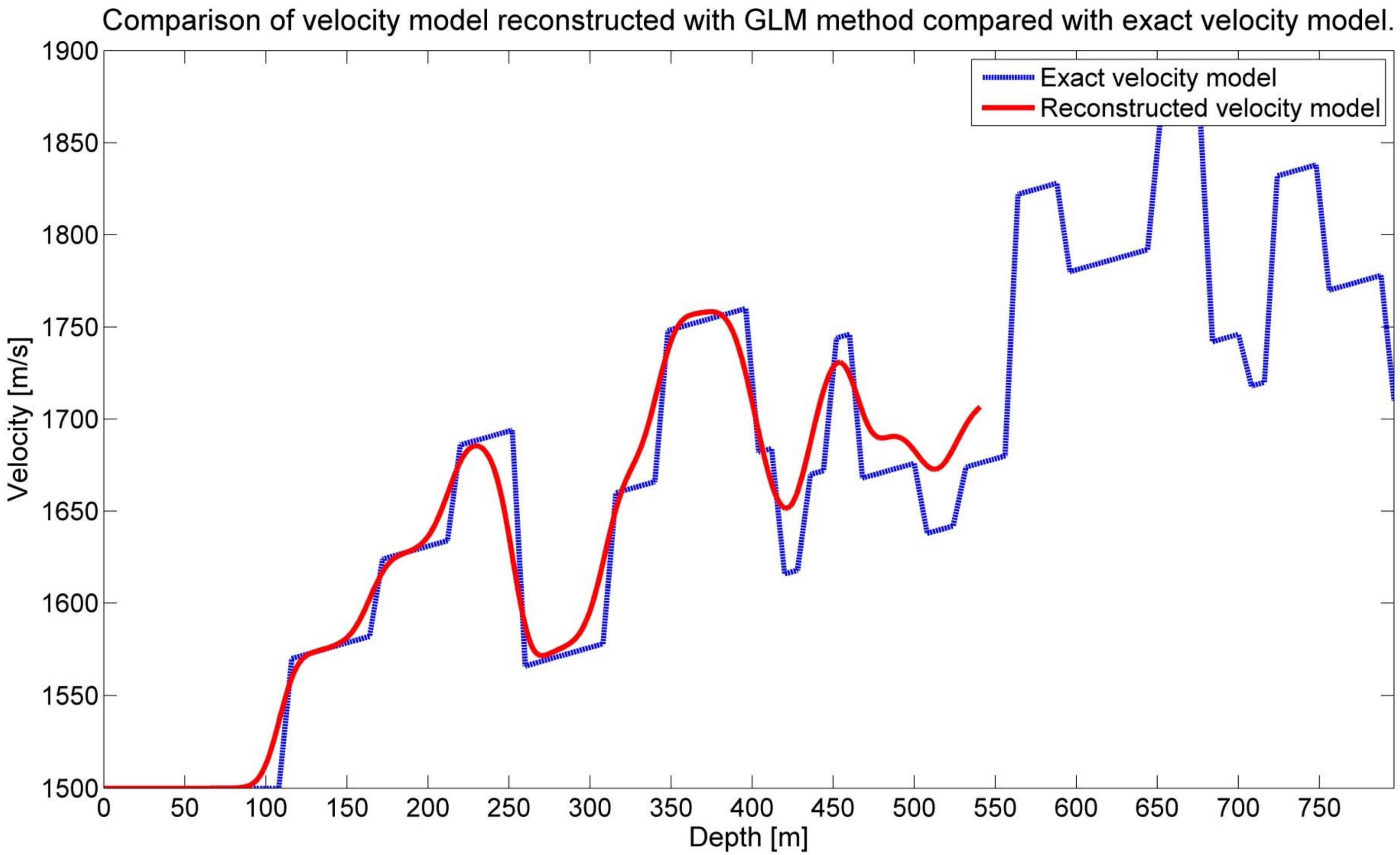
Numerical results for 2D



Numerical results for 2D



Numerical results for 2D



Conclusions

✓ GLM method is introduced for reconstructing of layered horizontally homogeneous velocity model in 1D and 2D cases

✓ Method is proved to be efficient and accurate for smooth velocity distributions

- Rather low noise level affects the results dramatically

Future work

- Expansion to 3D media
- Improvement of the technique for the noisy data
- Development and implementation of the theory for slightly horizontally inhomogeneous media
- Investigation of the applicability of the method to the construction of the reference velocity model
- Comparison with other inverse method techniques

References

1. Gelfand, I.M. and Levitan, B.M. [1951] Reconstruction of the differential equation by its spectral function. Izv. Akad. Nauk. USSR, Ser. Mat., 15(4), 309-360 (in russian).

2. Kay, I. and Moses, N.E. [1956] The determination of the scattering potential from spectral measure function. Nuovo Cimento, 3(2), 276-304.

3. Blagovestchenskii, A.S. [2001] Inverse Problems of Wave Processes. V.S.P. The Netherlands.

Acknowledgements

I'm grateful to Alexandr Blagovestchenskii for immense support in theoretical understanding

Thank you for your attention!

JASS 2009