Even degree surfaces

Joint Advanced Student School

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Agenda

- Introduction
- Quadrics and their kinematical description
- Superquadrics
  - Definition
  - Examples
  - Applications
- Conclusion

Introduction

- Disadvantages of polygonal-based technologies:
  - Low precision
  - Number of polygons depends on distance from an object
  - Lots of “garbage”(degenerate polygons)
- Analytical models solve these problems
- Need of classification of higher degree surfaces

→ Superquadrics

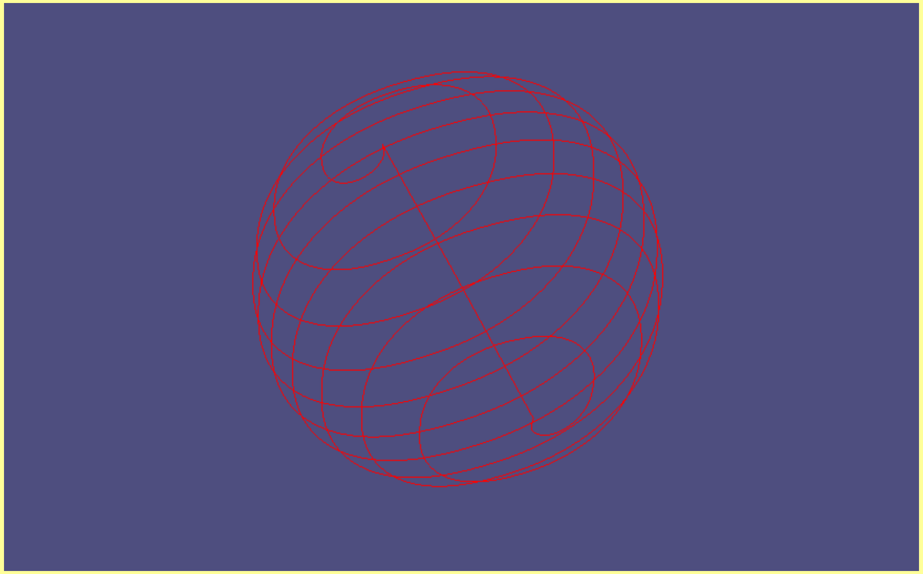
Kinematical Quadrics

- Parametrical description of quadrics
- Description of quadrics based on spatial spiral
  - Sphere(ellipsoid)
  - Hyperboloid
  - Cone
  - Paraboloid

Kinematical quadrics

- Ellipsoid

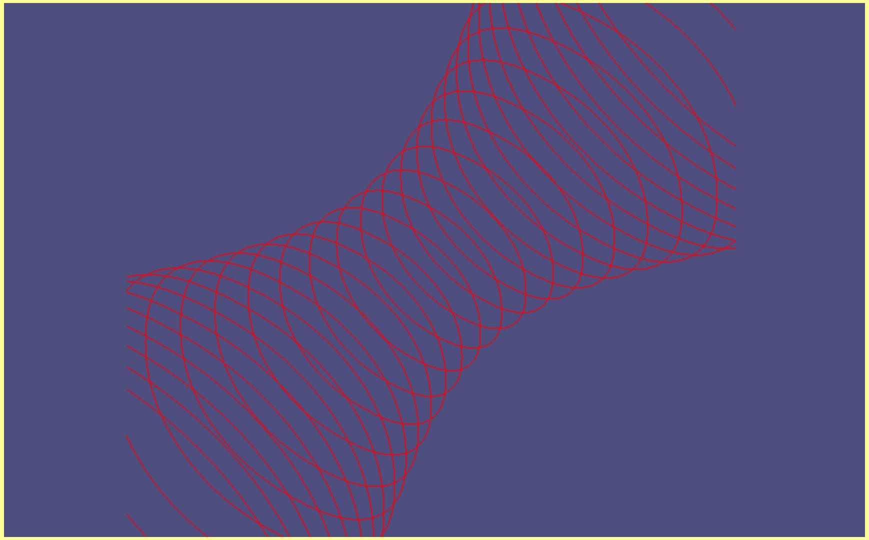
$$\begin{cases} x = \sqrt{(r^2 - z^2)} \times \sin(z*step), \\ y = \sqrt{(r^2 - z^2)} \times \cos(z*step), \\ Z1 <= z <= Z2 \end{cases}$$



Kinematical quadrics

- Hyperboloid of one sheet

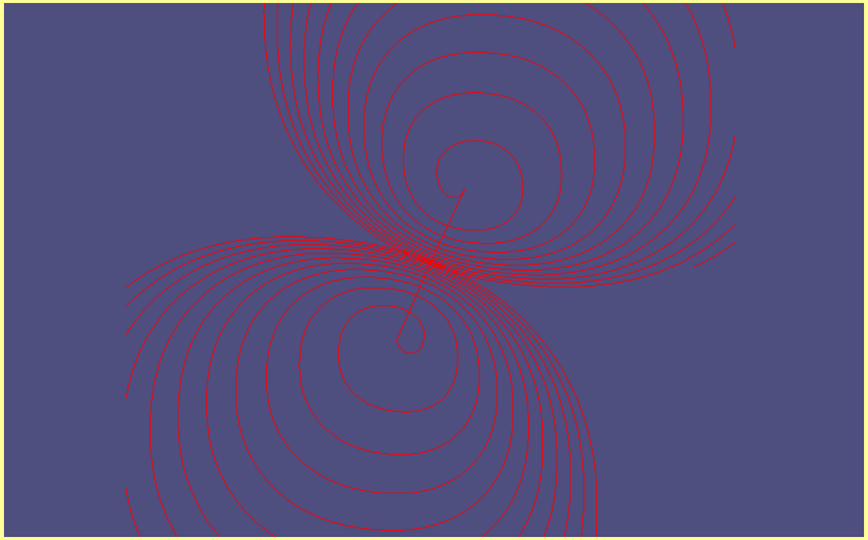
$$\begin{cases} x = \sqrt{(r^2+z^2)} \times \sin z, \\ y = \sqrt{(r^2+z^2)} \times \cos z, \\ Z1 <= z <= Z2 \end{cases}$$



Kinematical quadrics

- Hyperboloid of two sheets

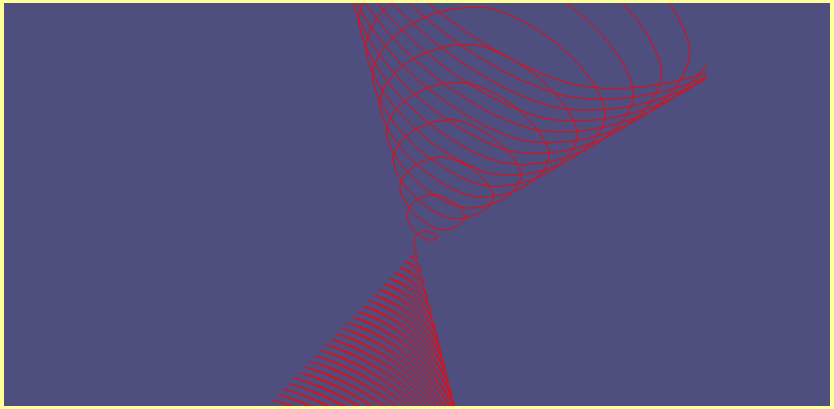
$$\begin{cases} x = \sqrt{(-r^2+z^2)} \times sin\ (z*step), \\ y = \sqrt{(-r^2+z^2)} \times cos\ (z*step), \\ Z1 <= z <= Z2 \end{cases}$$



Kinematical quadrics

- Cone

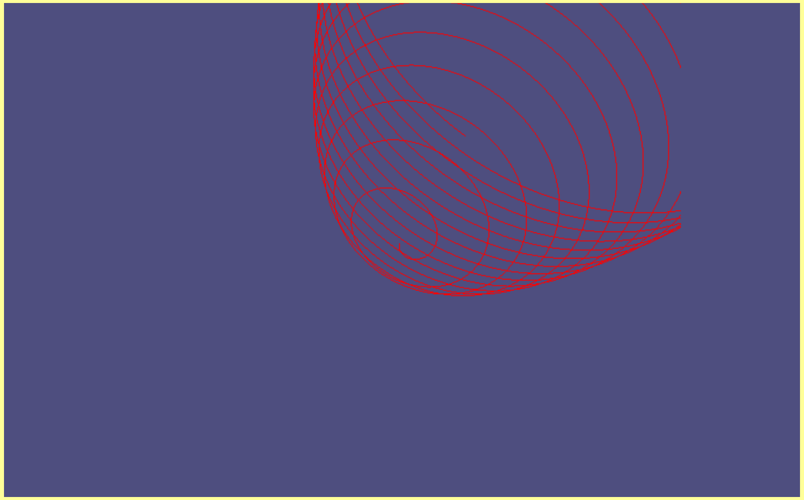
$$\begin{cases} x = z \times \sin(z*step), \\ y = z \times \cos(z*step), \\ Z1 <= z <= Z2 \end{cases}$$



Kinematical quadrics

- Paraboloid

$$\begin{cases} x = \sqrt{z} \times \sin z, \\ y = \sqrt{z} \times \cos z, \\ Z1 <= z <= Z2 \end{cases}$$



Kinematical Superquadrics

- Superquadrics are a generalization of quadrics, being very flexible 3-dimensional parametric objects.
- Mathematical representation of Superquadric is very simple
- By adjusting a relatively few number of parameters of a superquadric, a large variety of shapes may be obtained.
- Term “Superquadric” was defined by Barr in *“Superquadrics and angle preserving transformations”(*1981)

Kinematical Superquadrics

- Description of kinematical superquadrics with spatial spirals
- Classification is similar to given classification of quadrics
  - Only sine and cosine functions are used
  - Commonly:

    $x = \mathit{form} * \cos^p(z * step)$

    $y = \mathit{form} * \sin^p(z * step)$

    $Z1 <= z <= Z2$

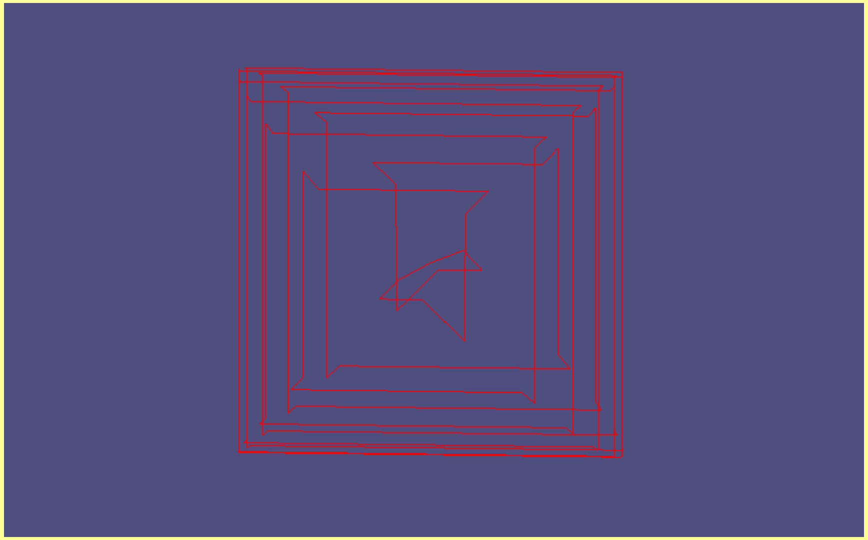
Examples of superquadrics

- Sphere(ellipsoid)
- Hyperboloid
- Cone
- Paraboloid

Kinematical superquadrics

- Ellipsoid

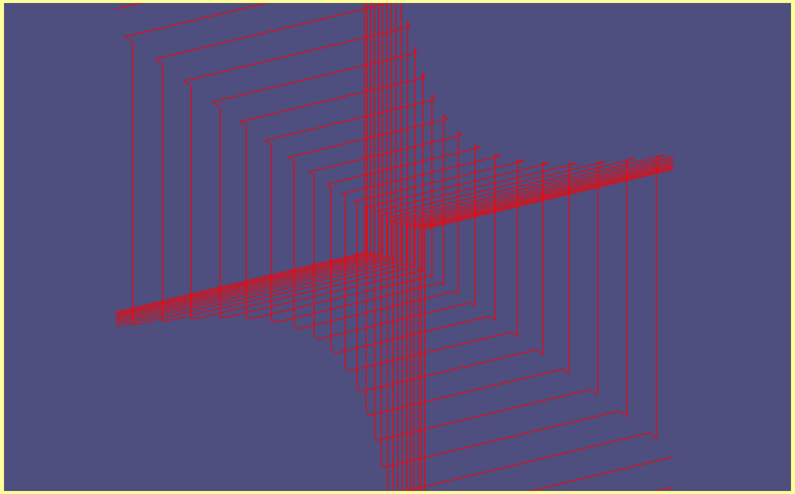
$$\begin{cases} x = \sqrt{(r^2 - z^2)} \times \sin^p z, \\ y = \sqrt{(r^2 - z^2)} \times \cos^p z, \\ Z1 <= z <= Z2 \end{cases}$$



Kinematical superquadrics

- Hyperboloid of one sheet

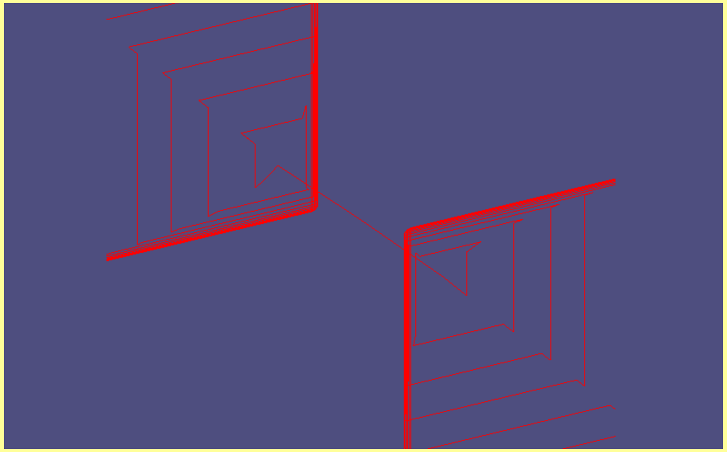
$$\begin{cases} x = \sqrt{(r^2+z^2)} \times \sin^p z, \\ y = \sqrt{(r^2+z^2)} \times \cos^p z, \\ Z1 <= z <= Z2 \end{cases}$$



Kinematical superquadrics

- Hyperboloid of two sheets

$$\begin{cases} x = \sqrt{(-r^2+z^2)} \times \sin^p (z*step), \\ y = \sqrt{(-r^2+z^2)} \times \cos^p (z*step), \\ Z1 <= z <= Z2 \end{cases}$$



Kinematical superquadrics

- Cone

$$\begin{cases} x = z \times \sin^p (z*step), \\ y = z \times \cos^p (z*step), \\ Z1 <= z <= Z2 \end{cases}$$

Kinematical Superquadrics

- Classification of superquadrics by Barr:

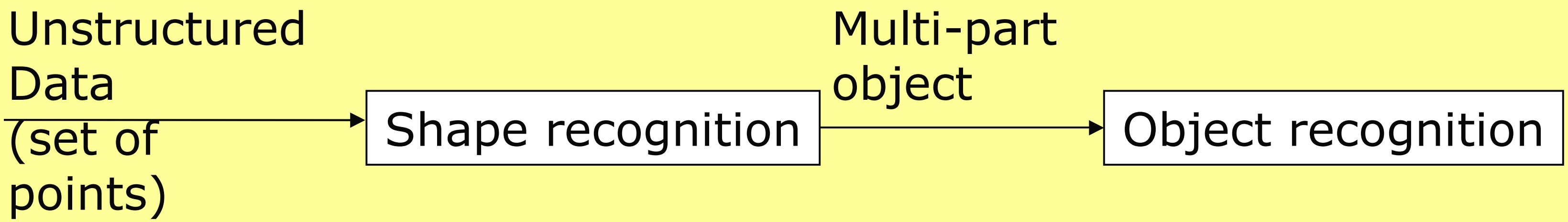
Type	Representation
Superellipsoid	$\underline{x}(\eta,\omega) = \begin{bmatrix} a_1 & \cos^{\varepsilon_1}\eta & \cos^{\varepsilon_2}\omega \\ a_2 & \cos^{\varepsilon_1}\eta & \sin^{\varepsilon_2}\omega \\ a_3 & \sin^{\varepsilon_1}\eta & \end{bmatrix}, \begin{matrix} -\pi/2 & \leq & \eta & \leq & \pi/2 \\ -\pi & \leq & \omega & < & \pi \end{matrix}$
Superhyperboloid of one sheet	$\underline{x}(\eta,\omega) = \begin{bmatrix} a_1 & \sec^{\varepsilon_1}\eta & \cos^{\varepsilon_2}\omega \\ a_2 & \sec^{\varepsilon_1}\eta & \sin^{\varepsilon_2}\omega \\ a_3 & \tan^{\varepsilon_1}\eta & \end{bmatrix}, \begin{matrix} -\pi/2 & < & \eta & < & \pi/2 \\ -\pi & \leq & \omega & < & \pi \end{matrix}$
Superhyperboloid of two sheets	$\underline{x}(\eta,\omega) = \begin{bmatrix} a_1 & \sec^{\varepsilon_1}\eta & \sec^{\varepsilon_2}\omega \\ a_2 & \sec^{\varepsilon_1}\eta & \tan^{\varepsilon_2}\omega \\ a_3 & \tan^{\varepsilon_1}\eta & \end{bmatrix}, \begin{matrix} -\pi/2 & < & \eta & < & \pi/2 & \\ -\pi/2 & < & \omega & < & \pi/2 & \text{sheet 1} \\ \pi/2 & < & \omega & < & 3\pi/2 & \text{sheet 2} \end{matrix}$
Supertoroid	

Applications of superquadrics

- Superquadrics can be used for:
  - Scene(object) recognition
  - Tracking of changing objects
  - Computer-aided design

Applications of superquadrics

- Scene recognition
  - Scene recognition:



  - During object recognition objects are matched with precalculated ones, stored in database → size of these objects is important → describing objects with superquadrics

Applications of superquadrics

- Scene recognition: Possible ways to break objects being recognized into superquadrics:
  - Region growing
  - Split and merge

Applications of superquadrics

- Tracking of changing objects
  - Can be used for tracking medical data

Applications of superquadrics

- Computer-aided design
  - Analytical models of superquadrics can be useful in computer-aided modeling, providing possibilities for both smooth and sharp joining of objects

Applications of superquadrics

- Computer-aided design
  - Main limitation by using superquadrics – they do not allow free-form modeling – can be removed through usage of following techniques:
    - Using superquadrics as field functions for blob models
    - Implementation of superquadric deformation techniques(but: it leads to change of the degree)

Applications of superquadrics

- Examples of transformation operations are
  - Tapering
  - Twisting
  - Bending

Applications of superquadrics: Computer-aided design

- Transformations: Tapering
  - Tapering function:

    $X = x$

    $Y = fy(x)*y$

    $Z = fz(x)*z$

    Where:

    $fy(x) = ty*x/ay + 1, -1<=ty<=1$

    $fz(x) = tz*x/az + 1, -1<=tz<=1$

Applications of superquadrics: Computer-aided design

- Transformations: Twisting
  - Twisting function:

$A = f(x) = n*\pi(1 + x/a)$

$X = x$

$Y = y\cos(A) - z\sin(A)$

$Z = y\sin(A) + z\cos(A)$

n– number of twists

Applications of superquadrics: Computer-aided design

- Transformations: Bending
  - Bending function:

$A = ky$

$X = x$

$Y = -\sin(A)(z-1/k)$

$Z = \cos(A)(z-1/k) + 1/k$

$1/k$– radius of the curvature

Conclusion

- Thanks for your attention!
- Any questions?