Sensor Fusion: Particle Filter

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Outline

- Motivation
- Applications
- Fundamentals
- Tracking People
- Advantages and disadvantages
- Summary

Problem Statement

- **Tracking the state** of a system as it evolves over **time**
- We have: Sequentially arriving (noisy or ambiguous) **observations**
- We want to know: Best possible **estimate** of the hidden variables

Motivation

- The trend of addressing **complex problems** continues
- Large number of applications require evaluation of integrals
- Non-linear models
- Non-Gaussian noise

History

- First attempts – simulations of growing polymers
  - M. N. Rosenbluth and A.W. Rosenbluth, “Monte Carlo calculation of the average extension of molecular chains,” *Journal of Chemical Physics*, vol. 23, no. 2, pp. 356–359, 1956.
- First application in signal processing - 1993
  - N. J. Gordon, D. J. Salmond, and A. F. M. Smith, “Novel approach to nonlinear/non-Gaussian Bayesian state estimation,” *IEE Proceedings-F*, vol. 140, no. 2, pp. 107–113, 1993.
- Books
  - A. Doucet, N. de Freitas, and N. Gordon, Eds., *Sequential Monte Carlo Methods in Practice*, Springer, 2001.
  - B. Ristic, S. Arulampalam, N. Gordon, *Beyond the Kalman Filter: Particle Filters for Tracking Applications*, Artech House Publishers, 2004.
- Tutorials
  - M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, “A tutorial on particle filters for online nonlinear/non-gaussian Bayesian tracking,” *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 174–188, 2002.

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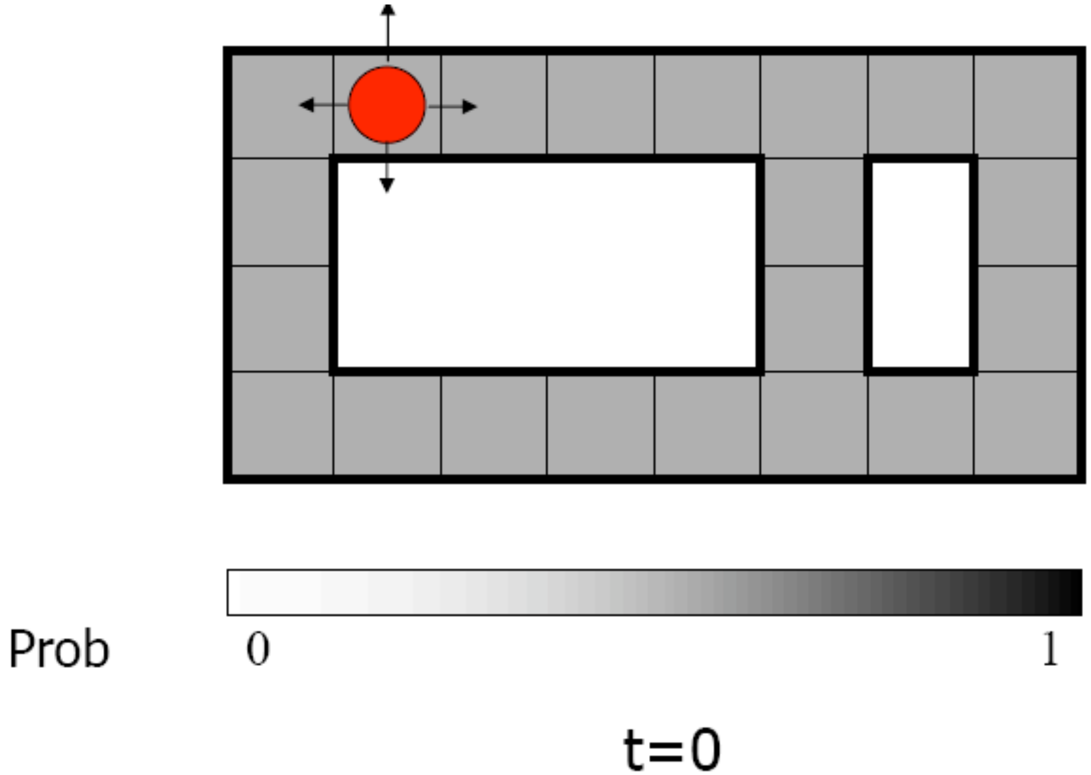
Application fields

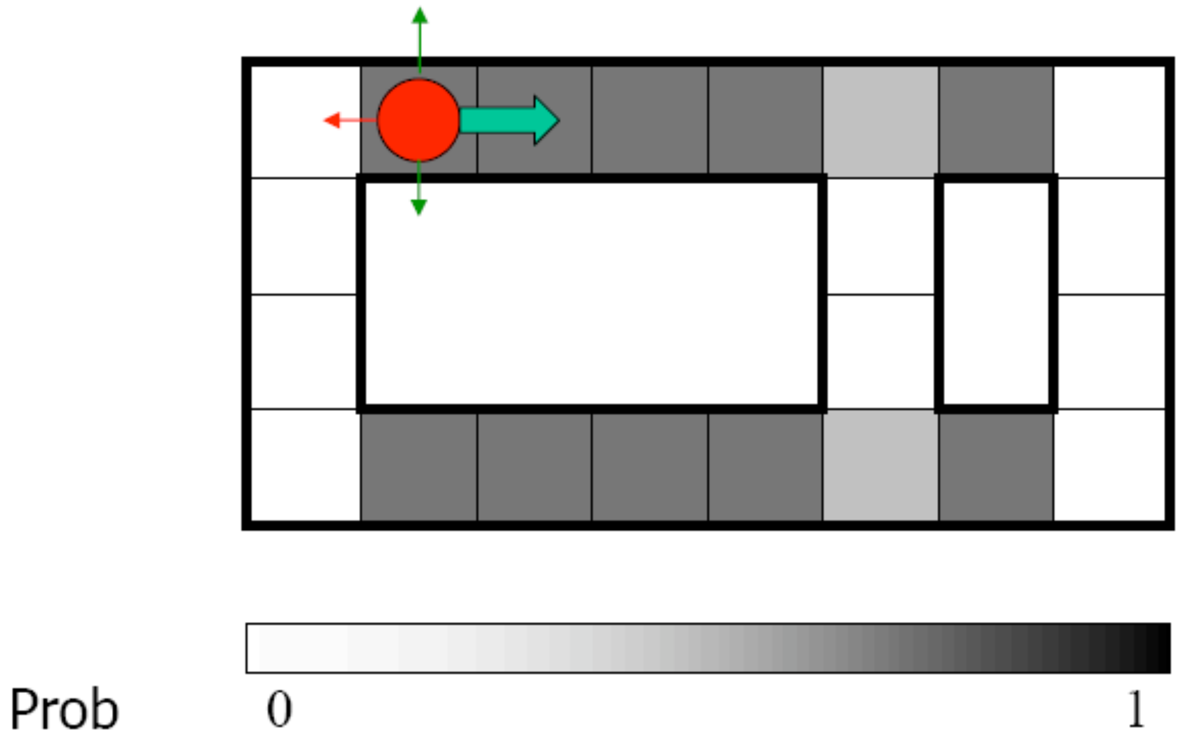
- **Signal processing**
  - Image processing and segmentation
  - Model selection
  - Tracking and navigation
- **Communications**
  - Channel estimation
  - Blind equalization
  - Positioning in wireless networks
- **Other applications**
  - Biology &Biochemistry
  - Chemistry
  - Economics & Business
  - Geosciences
  - Immunology
  - Materials Science
  - Pharmacology & Toxicology
  - Psychiatry/Psychology
  - Social Sciences

Example: Robot Localization

- Sensory model: never more than 1 mistake
- Motion model: may not execute action with small probability



Example: Robot Localization



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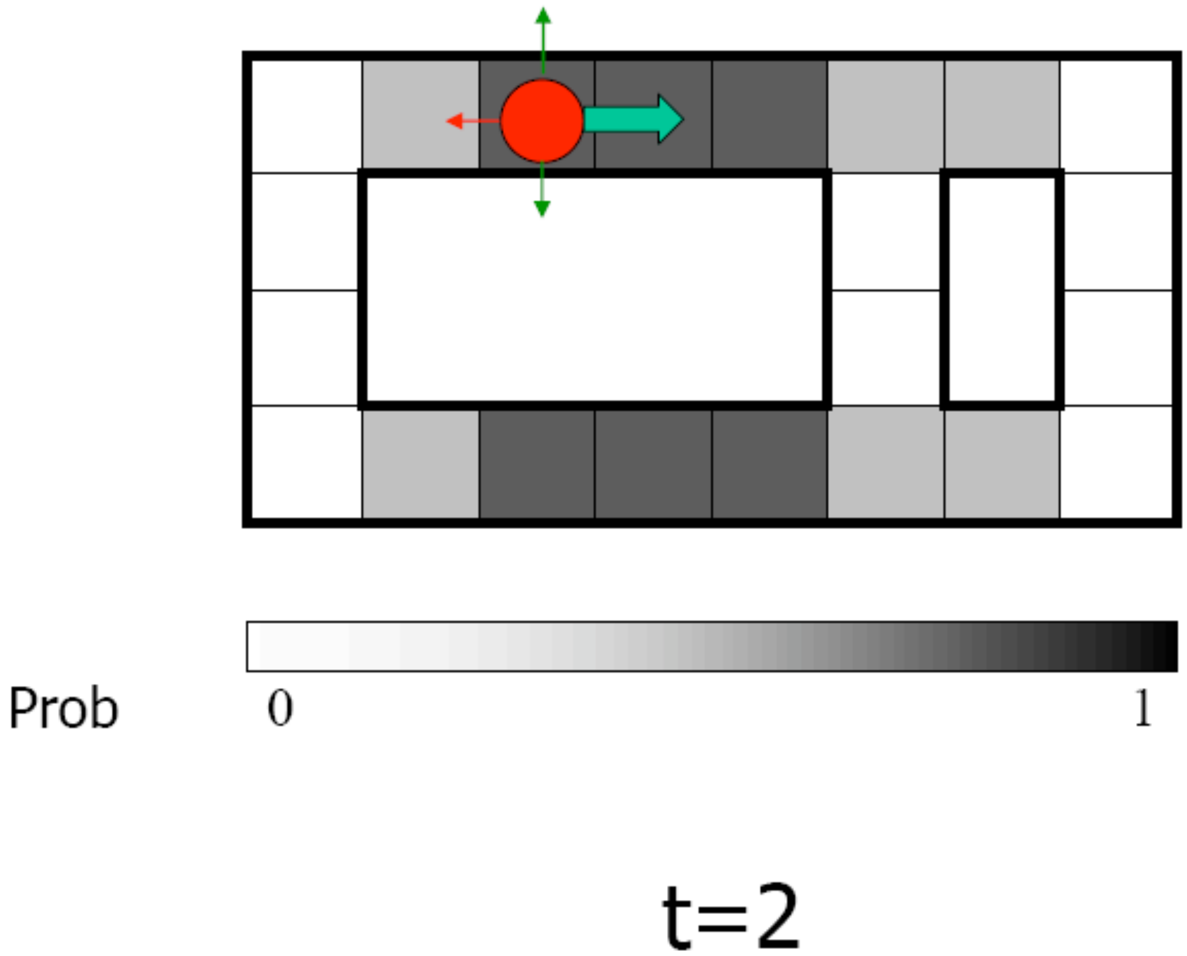
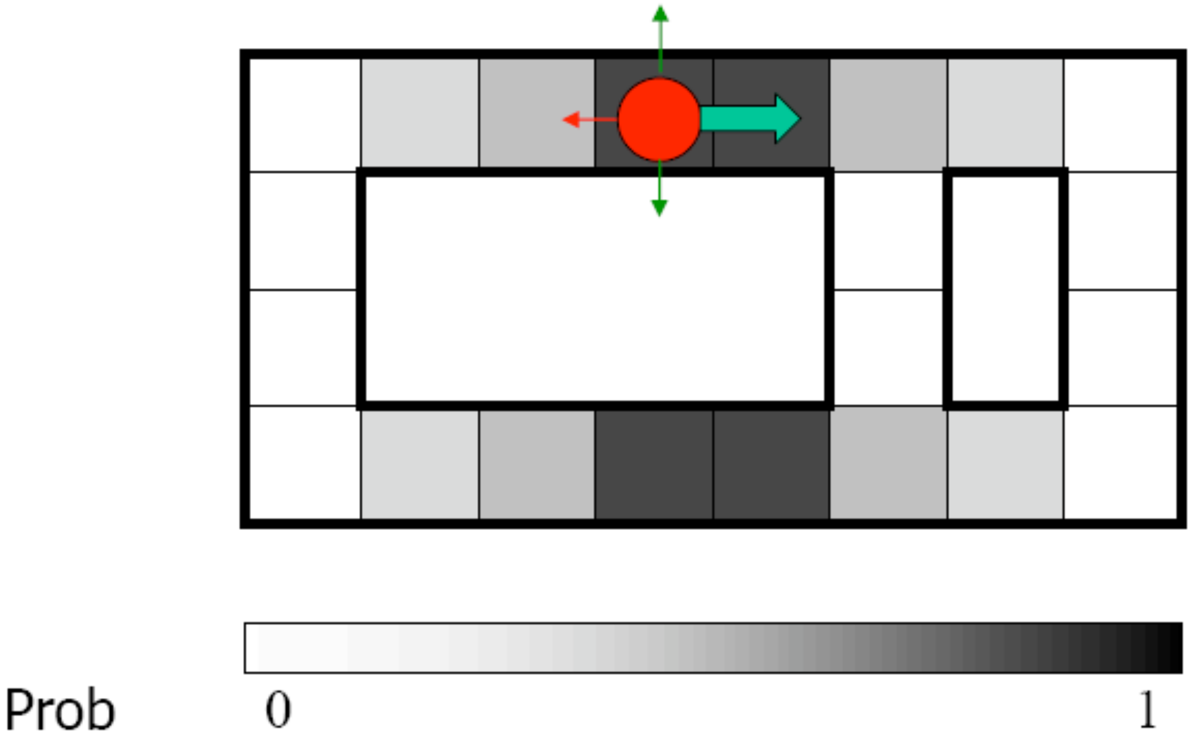
Example: Robot Localization



Example: Robot Localization



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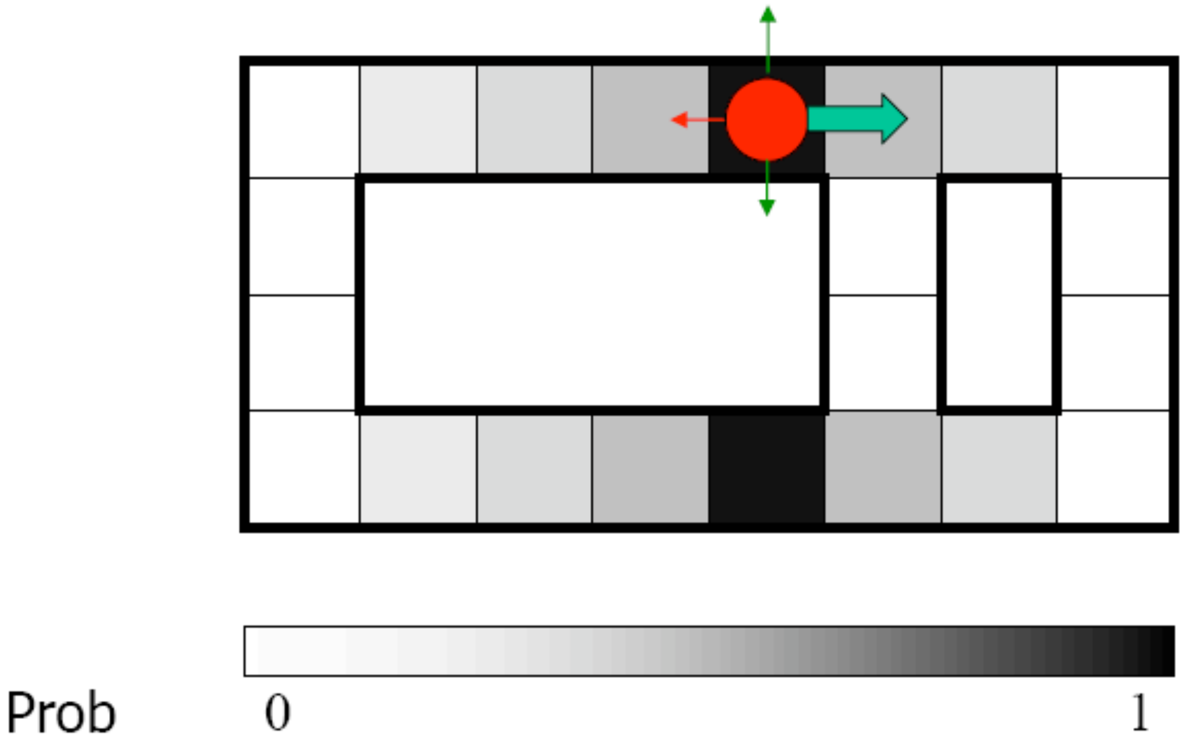
Example: Robot Localization



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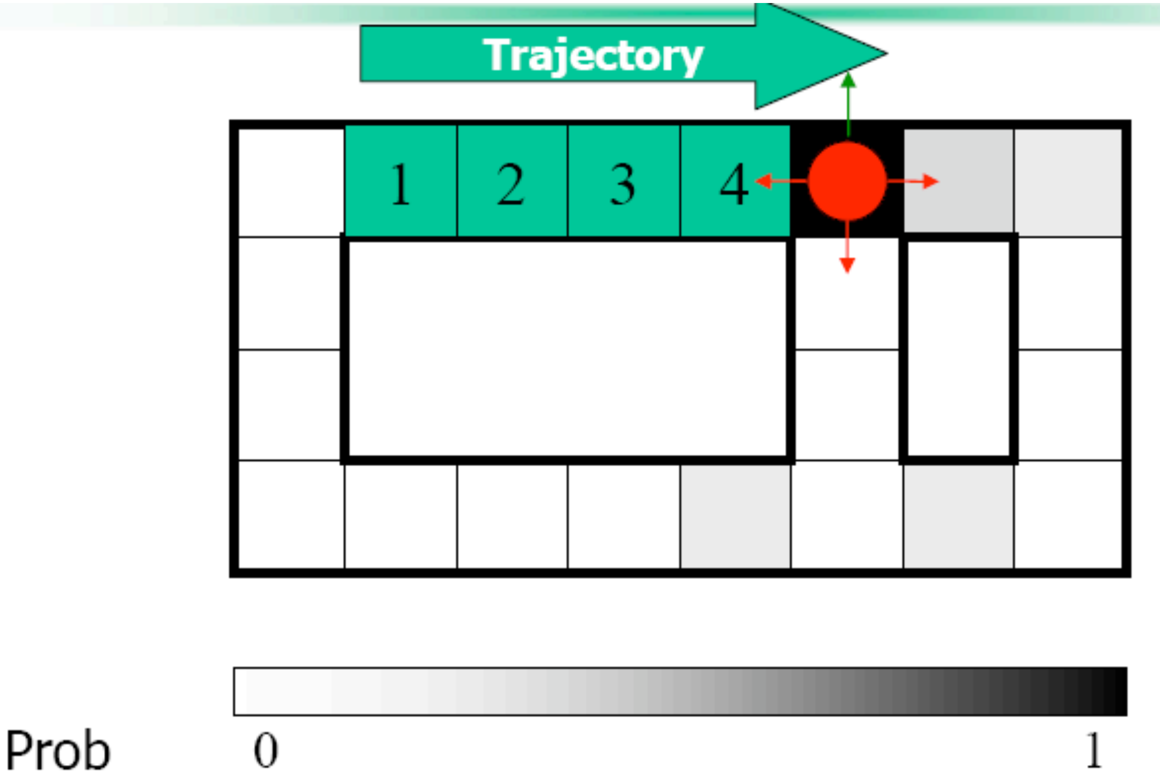
Example: Robot Localization



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Applications: Example

- Observations are the velocity and turn information[1)]
- A car is equipped with an electronic roadmap
- The initial position of a car is available with 1km accuracy
- In the beginning, the particles are spread evenly on the roads
- As the car is moving the particles concentrate at one place

1) Gustafsson et al., "Particle Filters for Positioning, Navigation, and Tracking," *IEEE Transactions on SP,* 2002

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Fundamentals

- The Dynamic System Model
  - states of a system and state transition equation; measurement equation
- Bayesian Filter Approach
  - estimation of the state; probabilistic modelling; Bayesian filter
- Optimal and Suboptimal Solutions
  - KF and Grid Filter; EKF, Particle Filter ...

The Dynamic System

Modeling: **State Transition** or **Evolution Equation**

$$x_k = f_k(x_{k-1}, \mathbf{u}_{k-1}, \mathbf{v}_{k-1})$$

Where:

- $f(\cdot, \cdot, \cdot)$: evolution function (possible non-linear)
- x_k, x_{k-1}: current and previous state
- $\mathbf{v}_{k-1}$: state noise (usually not Gaussian)
- $\mathbf{u}_{k-1}$: known input

Note: state only depends on previous state, i.e. first order Markov process

The Dynamic System

Modeling: **Measurement Equation**

$$z_k = h_k(x_k, \mathbf{u}_k, n_k)$$

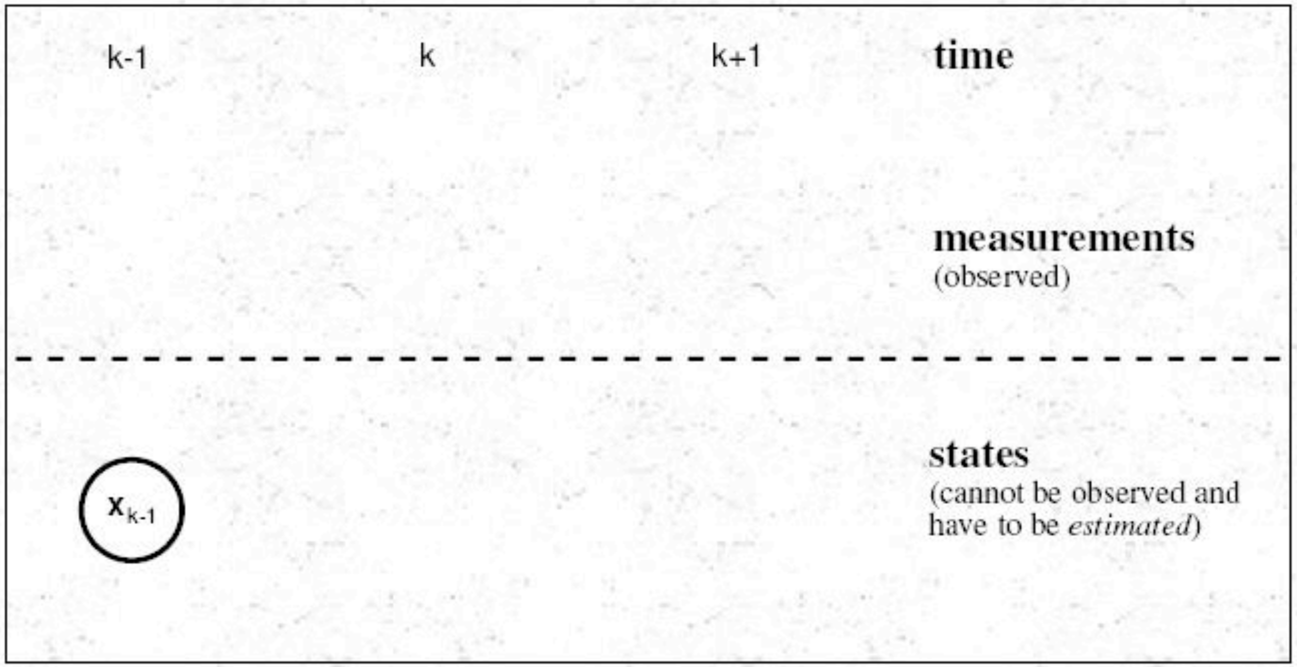
Where:

- ***h*** (·, ·, ·): measurement function (possible non-linear)
- $\mathbf{Z_k}$: measurement
- $\mathbf{n}_k$: measurement noise (usually not Gaussian)
- $\mathbf{u}_k$: known input

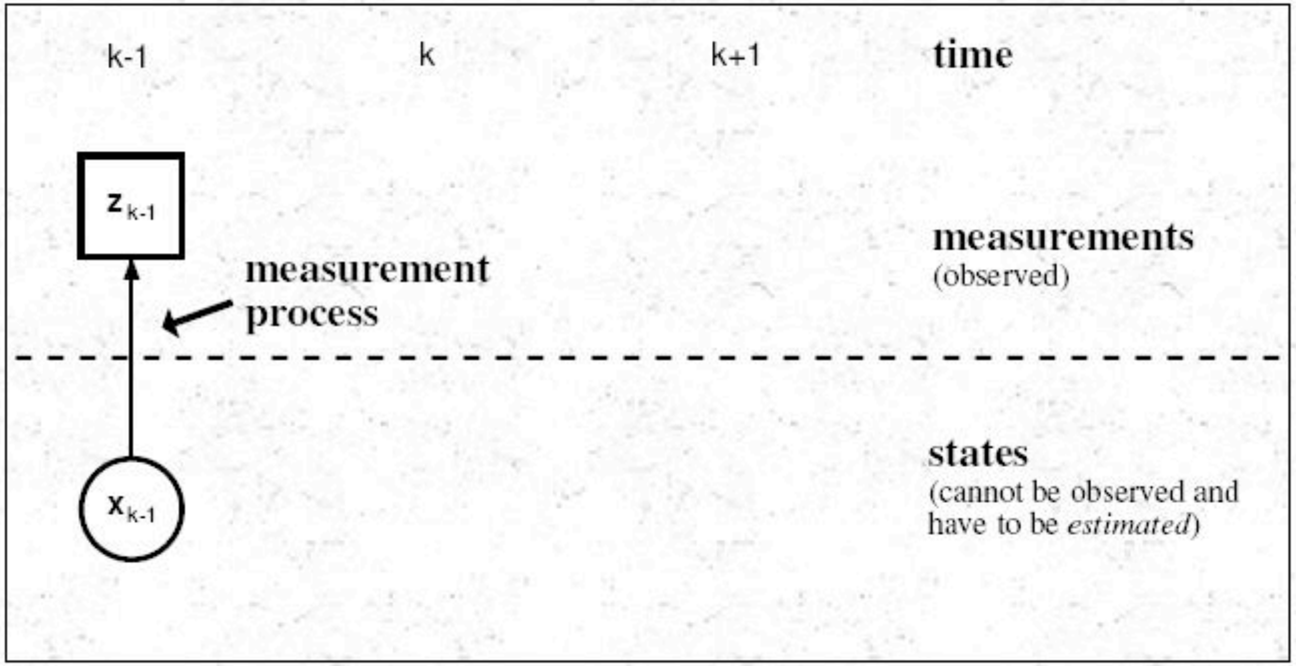
Remark:

- dimensionality of state, measurement, input, state noise, and measurement noise can all be different!)

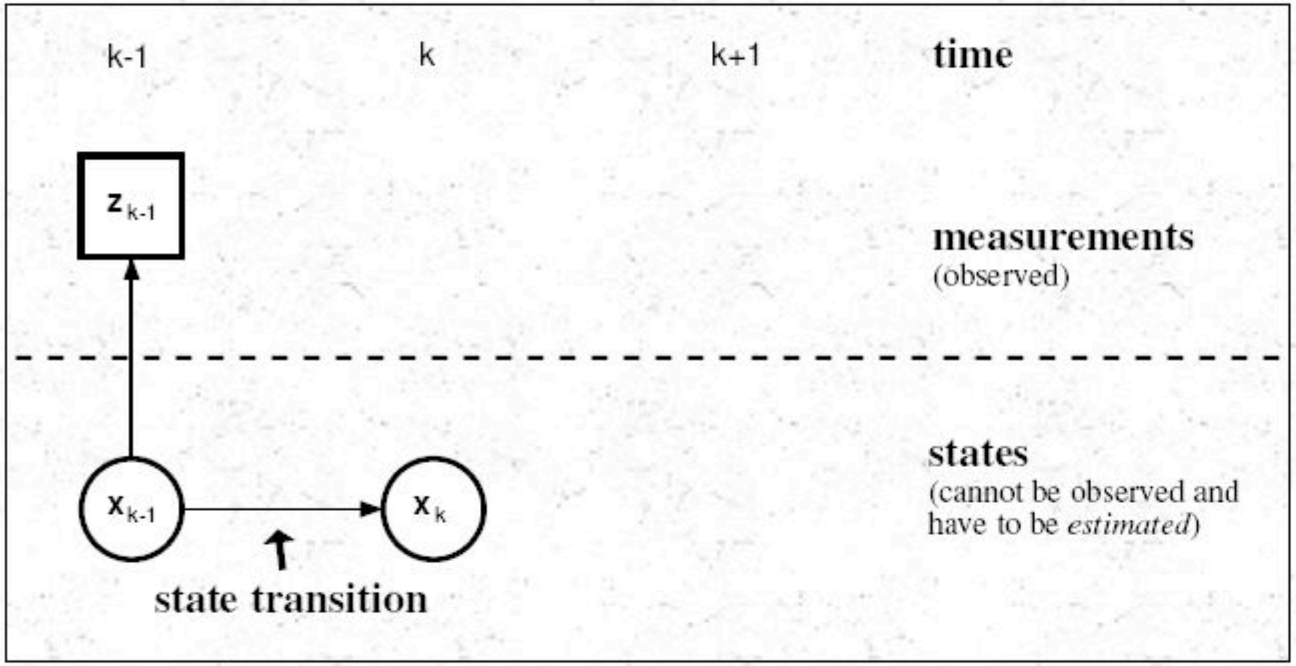
The Dynamic System



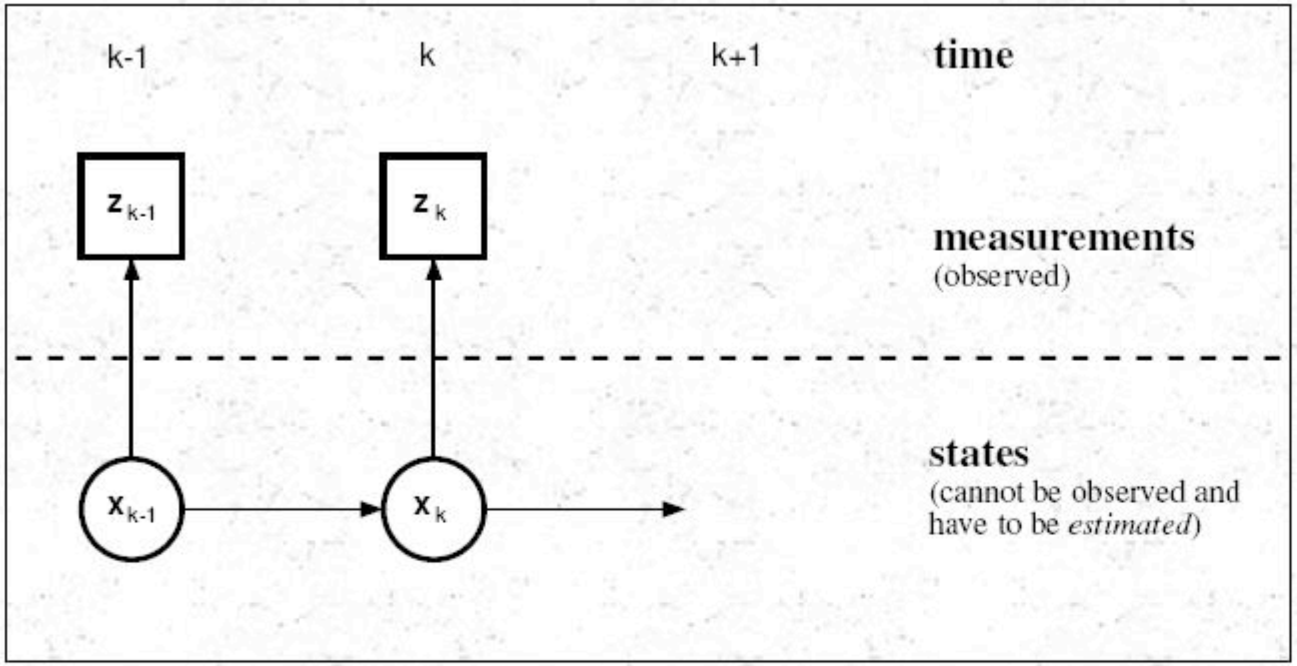
The Dynamic System



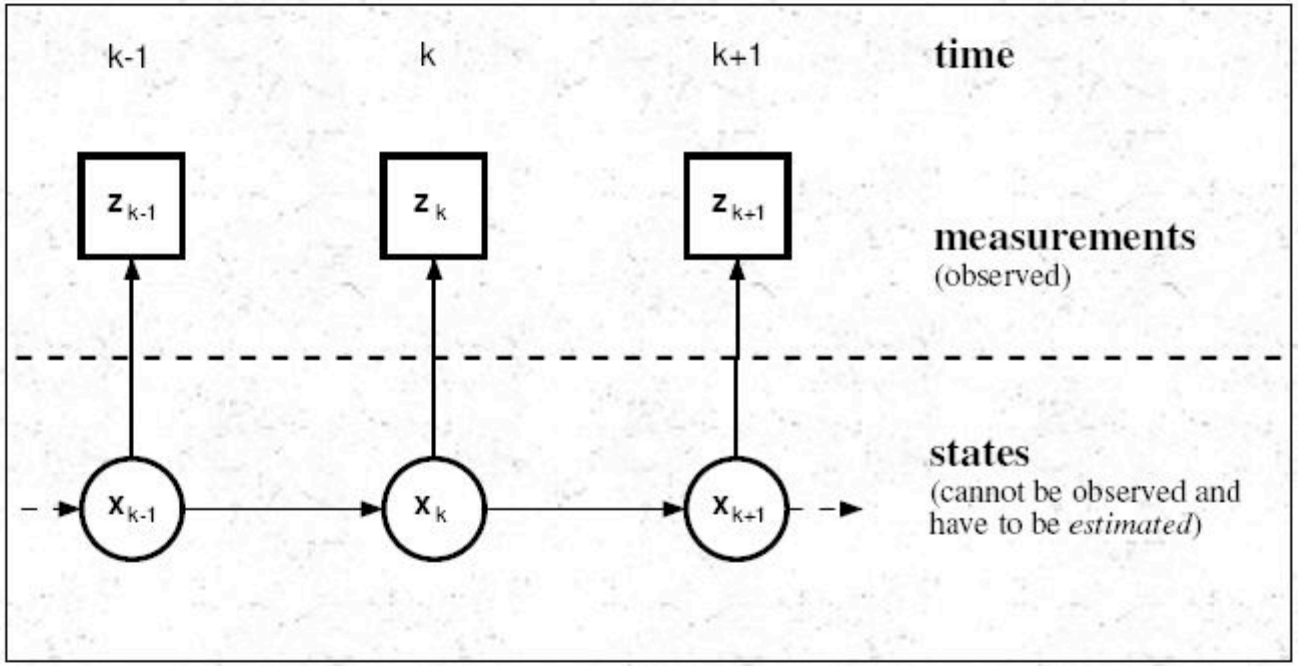
The Dynamic System



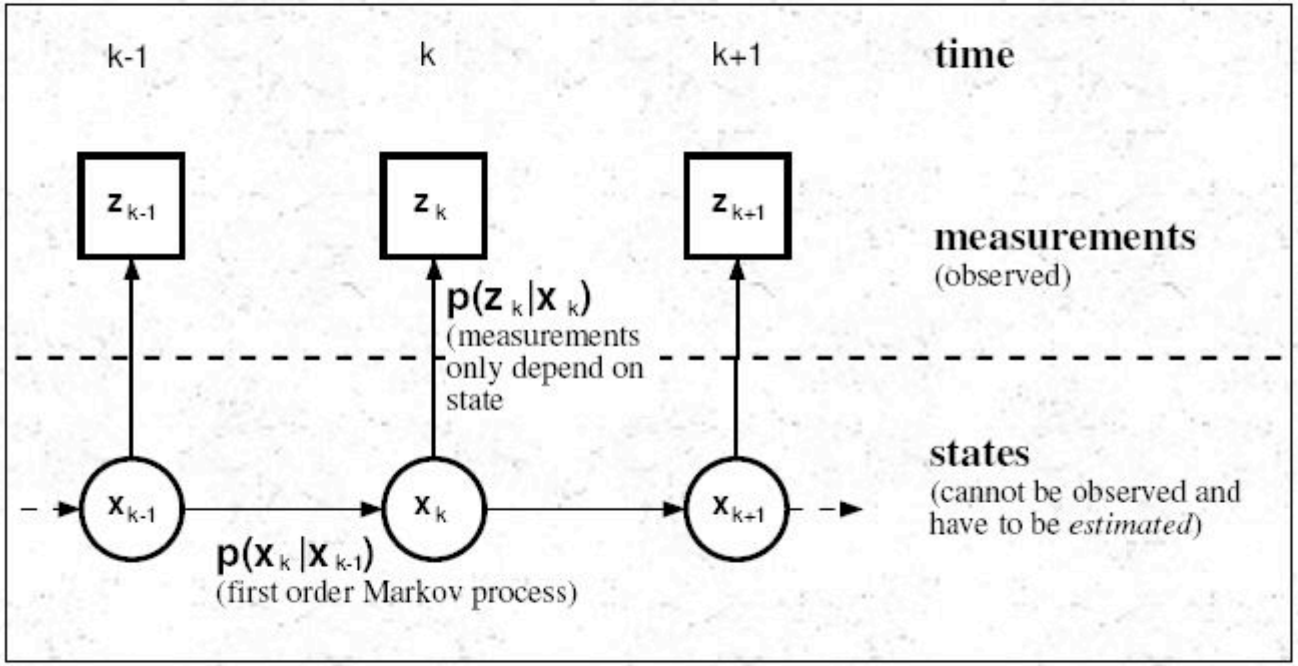
The Dynamic System



The Dynamic System



The Dynamic System



Bayesian Filtering-Tracking Problem

- Unknown State Vector $\mathbf{x}_{0:k} = (\mathbf{x}_0, \ldots, \mathbf{x}_k)$
- Observation Vector $\mathbf{z}_{1:k} = (\mathbf{z}_1, \ldots, \mathbf{z}_k)$
  - Find PDF $\mathbf{p}(\mathbf{x}_{0:k} \mid \mathbf{z}_{1:k})$ … posterior distribution
  - or $\mathbf{p}(\mathbf{x}_k \mid \mathbf{z}_{1:k})$ … filtering distribution
- Prior Information given:
  - $p(\mathbf{x}_0)$ … prior on state distribution
  - $\mathbf{p}(\mathbf{z}_k \mid \mathbf{x}_k)$ … sensor model
  - $\mathbf{p}(\mathbf{x}_k \mid \mathbf{x}_{k-1})$ … Markovian state-space model

Sequential Update

- Storing all incoming measurements is inconvenient
- Recursive filtering:
  - **Predict** next state pdf from current estimate
  - **Update** the prediction using sequentially arriving new measurements
- **Optimal** Bayesian solution: recursively calculating exact posterior density

Bayesian Filter Approach

- **Prediction** Stage: Chapman-Kolmogorov equation

$$p(\mathbf{x}_k|\mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{z}_{1:k-1})d\mathbf{x}_{k-1}$$

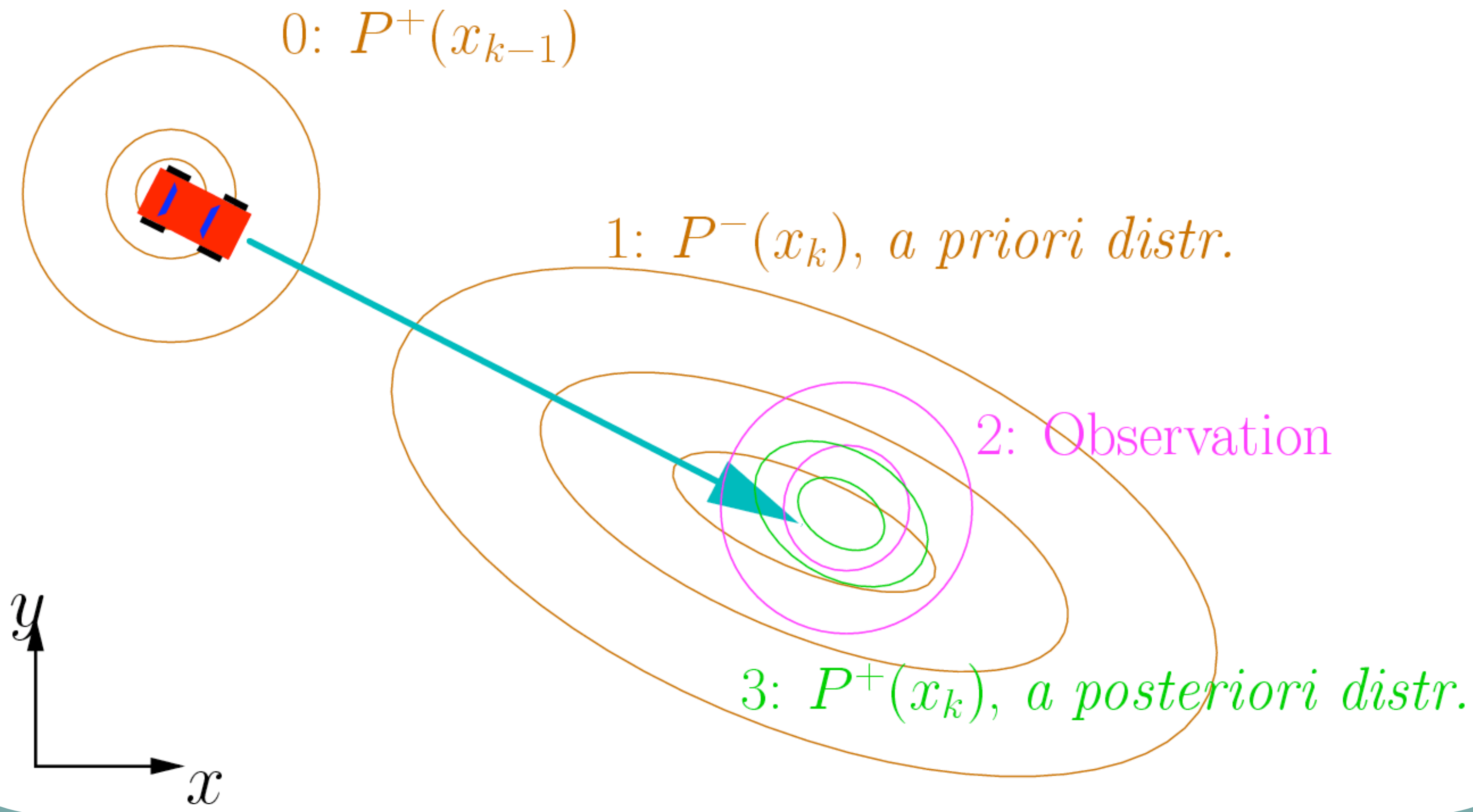
- **Update** Stage:

$$p(\mathbf{x}_k|\mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{z}_{1:k-1})}{p(\mathbf{z}_k|\mathbf{z}_{1:k-1})}$$

- BUT: This is **optimal** Bayesian Solution! For non-Gaussian there is no determined analytical solution
- Remedy: Approximation with EKF and **particle filter**

Bayesian Filter Approach

- Estimation Process



Reminder: Kalman Filter (KF)

- Optimal solution for linear-Gaussian case
- Assumptions:
  - State model is known linear function of last state and Gaussian noise signal
  - Sensory model is known linear function of state and Gaussian noise signal
  - Posterior density is Gaussian

Reminder: Limitations of KF

- Assumptions are „too strong“. We often find:
  - Non-linear Models
  - Non-Gaussian Noise or Posterior
  - Multi-modal Distributions
- Extended Kalman Filter:
  - local linearization of non-linear models
  - still limited to Gaussian posterior!

Particle Filter

- Different names:
  - (Sequential) Monte Carlo filters
  - Bootstrap filters
  - Condensation
  - Interacting Particle Approximations
  - Survival of the fittest

Particle Filter

- The key idea:
  - represent the required predictive or filtering distribution by a set of random samples (possibly with weights) and compute estimates

Particle Filter

- Two types of information required:
  - Data
    - Controls (e.g., robot motion commands) and
    - Measurements (e.g., camera images).
  - Probabilistic model of the system
- Data given by:
  - The measurement at time **t**: $z^t=(z_1, z_2, \ldots, z_t)$
  - The control asserted in the time interval **(t-1,t]**: $u^t=(u_1, u_2, \ldots, u_t)$
- Remark:
  - Superscript:denote all events leading up to time **t**
  - Subscript: event at time **t**

Probabilistic model of the system

- Particle filters, like any member of the family of Bayes filters such as KF, EKF, estimate the posterior distribution of the state of the dynamical system conditioned on the data $p(x_t \mid z^t, u^t)$

- Three probability distributions are required:
  - **A *measurement model, $p(z_t \mid x_t)$***
  - **A *control model, $p(x_t \mid u_t, x_{t-1})$***
  - **An *initial state distribution, $p(x_0)$***

- Represent the distribution using weighted samples

Particle Filter

- Definition:

  A set of random samples $\{X_{0:t}^{i}, w_{0:t}^{i}\}$ drawn from a distribution $q(x_{0:t}|z_{1:t})$ is said to be properly weighted with respect to $p(x_{0:t}|z_{1:t})$ if for any integrable function g() the following holds:

$$E_p(g(X_{0:t})) = \lim_{N\to\infty} \sum_{i=1}^{N} g(X_{0:t}^{(i)}) w_{0:t}^{(i)}$$

Particle Filter

- Random Measure $\{x_{0:k}^i, w_k^i\}$, i=1...N_s
- Posterior PDF $p(x_{0:k} \mid z_{1:k})$
- Set of support points $\{x_{0:k}^i$, i=1...$N_s\}$
- Assosiated weights $\{w_k^i$, i=1...$N_s\}$
- Then, pdf p() can be approximated by properly weighted samples (so called particles):

$$p(\mathbf{x}_{0:k}|\mathbf{z}_{1:k}) \approx \sum_{i=1}^{N_s} w_k^i \delta(\mathbf{x}_{0:k} - \mathbf{x}_{0:k}^i)$$

=> discrete weighted approximation to the true posterior $p(x_{0:k} \mid z_{1:k})$

Importance Sampling

- Suppose p(x)~π(x), π(x) can be evaluated
- Let $x^i \sim q(x)$, $i=1..N_s$, samples
  - q(x) - Importance Density
- Weighted approximation to density p():

$$p(x) \approx \sum_{i=1}^{N_s} w^i \delta(x - x^i)$$

where $w^i \propto \dfrac{\pi(x^i)}{q(x^i)}$ normalized weight of the i-th particle

Degeneracy Problem

- After a few iterations, all but one particle will have negligible weight
- Measure for degeneracy:

$$N_{eff} = \frac{N_s}{1 + \mathrm{Var}(w_k^{*i})}$$

$$w_k^{*i} = p(\mathbf{x}_k^i | \mathbf{z}_{1:k}) / q(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i, \mathbf{z}_k)$$

$$\widehat{N_{eff}} = \frac{1}{\sum_{i=1}^{N_s} (w_k^i)^2}$$

- Effective sample size
- Small N_{eff} indicates severe degeneracy
- Brute force solution: Use very large N

Particle Filtering Methods

- SIS-Method
  - Sequential Importance Sampling (Implementation of a recursive Bayesian filter wirh monte-carlo simulations)
- Other derived methods
  - Sequential Importance Resampling- **SIR**
  - Auxiliary SIR
  - Regularized Particle Filter

SIS Particle Filter: Algorithm

$$[\{\mathbf{x}_k^i, w_k^i\}_{i=1}^{N_s}] = \text{SIS}\ [\{\mathbf{x}_{k-1}^i, w_{k-1}^i\}_{i=1}^{N_s}, \mathbf{z}_k]$$

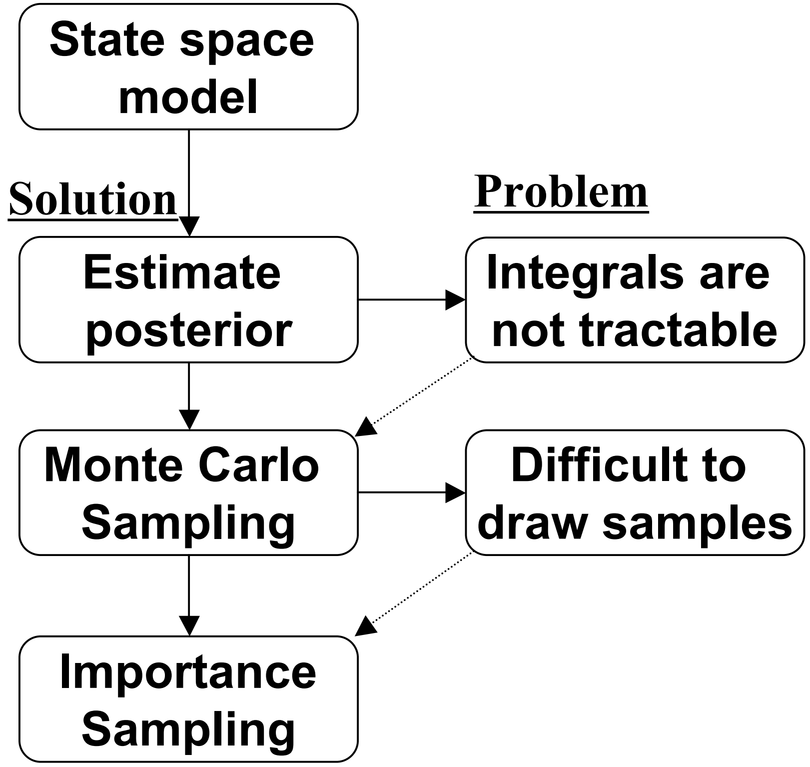
- FOR $i = 1 : N_s$
  - Draw $\mathbf{x}_k^i \sim q(\mathbf{x}_k | \mathbf{x}_{k-1}^i, \mathbf{z}_k)$
  - Assign the particle a weight, w_k^i
- END FOR

Where w^i_k

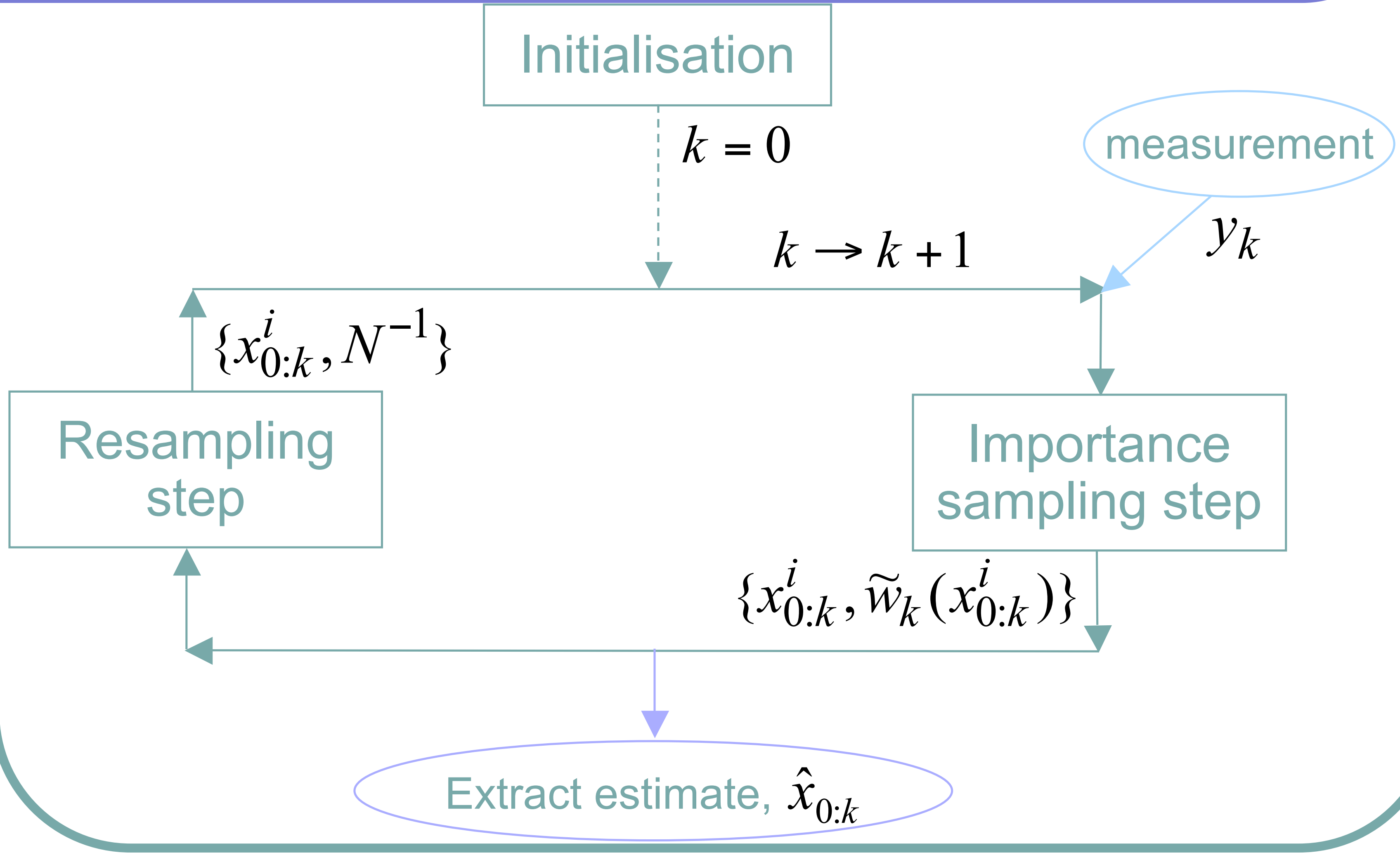
$$w_k^i \propto w_{k-1}^i \frac{p(\mathbf{z}_k | \mathbf{x}_k^i) p(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i)}{q(\mathbf{x}_k^i | \mathbf{x}_{k-1}^i, \mathbf{z}_k)}$$

SIS

- State space representation
- Bayesian filtering
- Monte-Carlo sampling
- Importance sampling



Basic Particle Filter - Schematic



SIS

- Degeneracy problem!
- Solutios:
  - Good choise of importance density (critical point!)

$$q(\mathbf{x}_k(\mathbf{x}_{k-1}^i, \mathbf{z}_k)_{opt} = p(\mathbf{x}_k(\mathbf{x}_{k-1}^i, \mathbf{z}_k)$$

$$= \frac{p(\mathbf{z}_k|\mathbf{x}_k(\mathbf{x}_{k-1}^i)p(\mathbf{x}_k(\mathbf{x}_{k-1}^i)}{p(\mathbf{z}_k(\mathbf{x}_{k-1}^i)}$$

$$w_k^i \propto w_{k-1}^i p(\mathbf{z}_k|\mathbf{x}_{k-1}^i)$$

$$= w_{k-1}^i \int p(\mathbf{z}_k|\mathbf{x}_k')p(\mathbf{x}_k'|\mathbf{x}_{k-1}^i)\,d\mathbf{x}_k'$$

  - Resampling

SIR Particle Filter: Algorithm

$[\{\mathbf{x}_k^i, w_k^i\}_{i=1}^{N_s}] = \text{SIR}\ [\{\mathbf{x}_{k-1}^i, w_{k-1}^i\}_{i=1}^{N_s}, \mathbf{z}_k]$

- FOR $i = 1 : N_s$
  - Draw $\mathbf{x}_k^i \sim p(\mathbf{x}_k|\mathbf{x}_{k-1}^i)$
  - Calculate $w_k^i = p(\mathbf{z}_k|\mathbf{x}_k^i)$
- END FOR
- Calculate total weight: $t = \text{SUM}\ [\{w_k^i\}_{i=1}^{N_s}]$
- FOR $i = 1 : N_s$
  - Normalise: $w_k^i = t^{-1} w_k^i$
- END FOR
- Resample using algorithm
  - $[\{\mathbf{x}_k^i, w_k^i, -\}_{i=1}^{N_s}] = \text{RESAMPLE}\ [\{\mathbf{x}_k^i, w_k^i\}_{i=1}^{N_s}]$

SIR Particle Filter: Algorithm

$[\{\mathbf{x}_k^{j*}, w_k^j, i^j\}_{j=1}^{N_s}] = \text{RESAMPLE}\ [\{\mathbf{x}_k^i, w_k^i\}_{i=1}^{N_s}]$

- Initialise the CDF: $c_1 = 0$
- FOR $i = 2 : N_s$
  - Construct CDF: $c_i = c_{i-1} + w_k^i$
- END FOR
- Start at the bottom of the CDF: $i = 1$
- Draw a starting point: $u_1 \sim \mathbb{U}\,[0, N_s^{-1}]$
- FOR $j = 1 : N_s$
  - Move along the CDF: $u_j = u_1 + N_s^{-1}(j-1)$
  - WHILE $u_j > c_i$
    - $i = i + 1$
  - END WHILE
  - Assign sample: $\mathbf{x}_k^{j*} = \mathbf{x}_k^i$
  - Assign weight: $w_k^j = N_s^{-1}$
  - Assign parent: $i^j = i$
- END FOR

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Tracking People

- Use of particle filters neccesary
- Two components:
  - Motion model (strong or weak)
  - Likelihood model (almost alwaus the most dificult part)

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Advantages

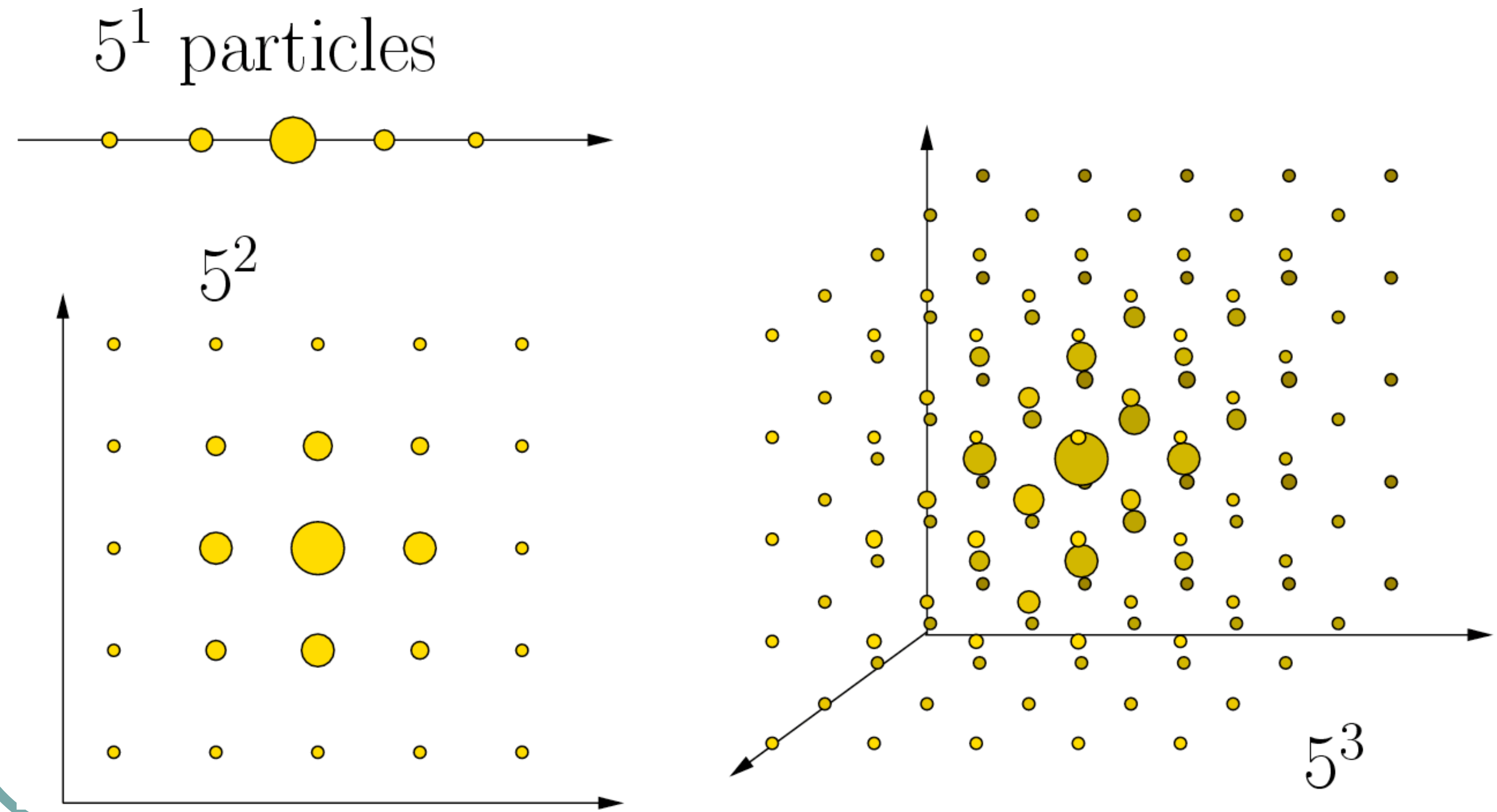
+ Ability to represent arbitrary densities
+ Adaptive focusing on probable regions of state-space
+ Dealing with non-Gaussian noise
+ The framework allows for including multiple models (tracking maneuvering targets)

Disadvantages

- High computational complexity
- It is difficult to determine optimal number of particles
- Number of particles increase with increasing model dimension
- Potential problems: degeneracy and loss of diversity
- The choice of importance density is crucial

Disadvantages

Number of particles grows exponentially with dimensionality of state space!



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Summary

- Particle Filters is an evolving and active topic, with good potential to handle “hard” estimation problems, involving non-linearity and multi-modal distributions.
- In general, the scheme is computationally expensive as the number of “particles” *N* needs to be large for precise results.
- Additional work required: optimizing the choice of *N*, and related error bounds.

Thank You for Your Attention!

Questions...?!